Physical Computation Workshop
Irish College, Leuven, 3-4 September 2025

KU LEUVEN

Reconciling Physics and Algorithmics

Is digital artificial intelligence (AI) about to launch a new age for humanity, or is it instead reaching the limits of its potential? The debate is raging everywhere, from the most expert circles to social media and daily news. It is a historical fact of all empires that the early signs of their declines coincide with a messianic tendency. For sure, the digital AI industry has become an empire. And for sure, there are accumulating signs that the model it relies upon is becoming more vulnerable, unsustainable, and exposed to its inherent limitations by the day.

What might announce the fall of an empire often lies in what enabled its rise, which for digital technology is pretty clear: the separation between physics and algorithmics. Before the invention of the computer, the algorithms of machines were analog, that is, physical

Physics of Computation Conference Endicott House MIT May 6-8, 1981

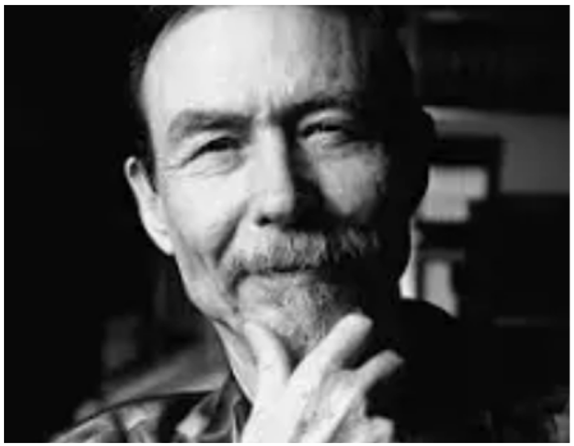
1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

of control is a fruit of their encounter. Without algorithmics, control is only half of control. It inherits the nobility of mathematical physics but does not

Mead, and John Hopfield. Three years later, the course split into three separate courses: Richard Feynman had launched quantum computing, Cary-

Physical computation

Carver Mead



Neuromorphic computing

A machine that awaits a theory

Richard Feynman

Quantum computing

A theory that awaits a machine

John Hopfield

Collective computing

A driving paradigm

Physical computation

Can physics solve the digital crisis ?

Why does it pay off to acknowledge the physics of a machine ?

Is there any relevant connection between quantum computing, Ising computing, and neuromorphic computing ?

What are the strengths and limitations of the respective computing machines ?

Physical modelling at scale

Rodolphe Sepulchre
Joint work with Fulvio Forni

Physical Computation Workshop
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Contents

- Motivation: the complexity of neuronal modelling
- Inspiration: signal modelling at scale
- Proposal: physical modelling at scale
- Result: neuronal modelling at scale

Motivation

Can physical modeling help taming the complexity of neuronal modelling ?

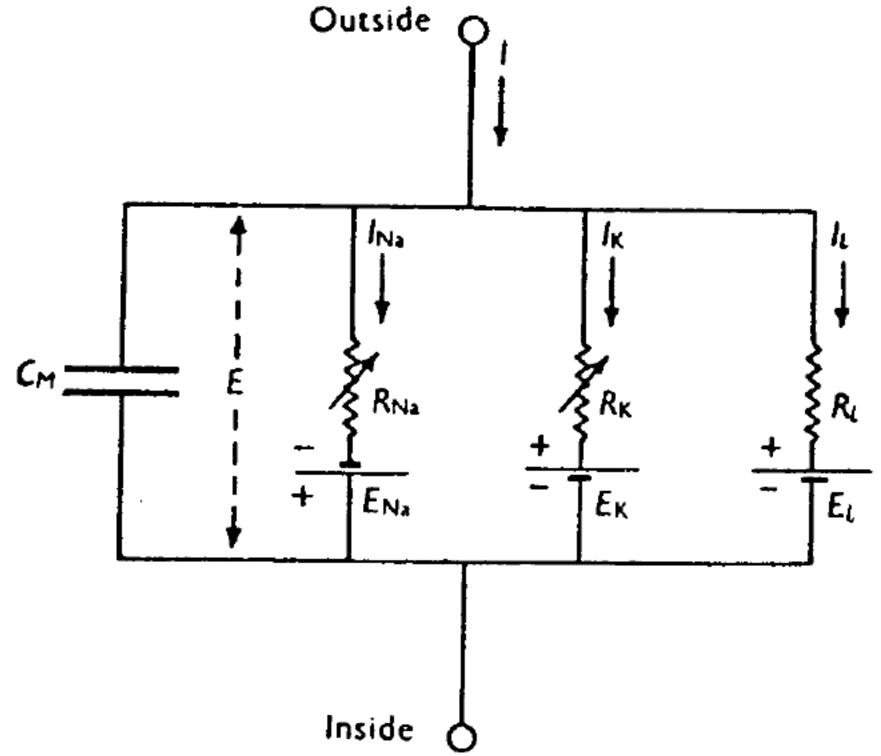


Fig. 1. Electrical circuit representing membrane. $R_{Na}=1/g_{Na}$; $R_K=1/g_K$; $R_l=1/\bar{g}_l$. R_{Na} and R_K vary with time and membrane potential; the other components are constant.

J. Physiol. (1952) 117, 500–544

A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE

By A. L. HODGKIN and A. F. HUXLEY

From the Physiological Laboratory, University of Cambridge

Hodgkin Huxley model

Summary of equations and parameters

We may first collect the equations which give the total membrane current I as a function of time and voltage. These are:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_l (V - V_l), \tag{26}$$

where

$$dn/dt = \alpha_n(1-n) - \beta_n n, \tag{7}$$

$$dm/dt = \alpha_m(1-m) - \beta_m m, \tag{15}$$

$$dh/dt = \alpha_h(1-h) - \beta_h h, \tag{16}$$

and

$$\alpha_n = 0{\cdot}01\,(V+10) \Big/ \left(\exp\frac{V+10}{10} - 1\right), \tag{12}$$

$$\beta_n = 0{\cdot}125\,\exp\,(V/80), \tag{13}$$

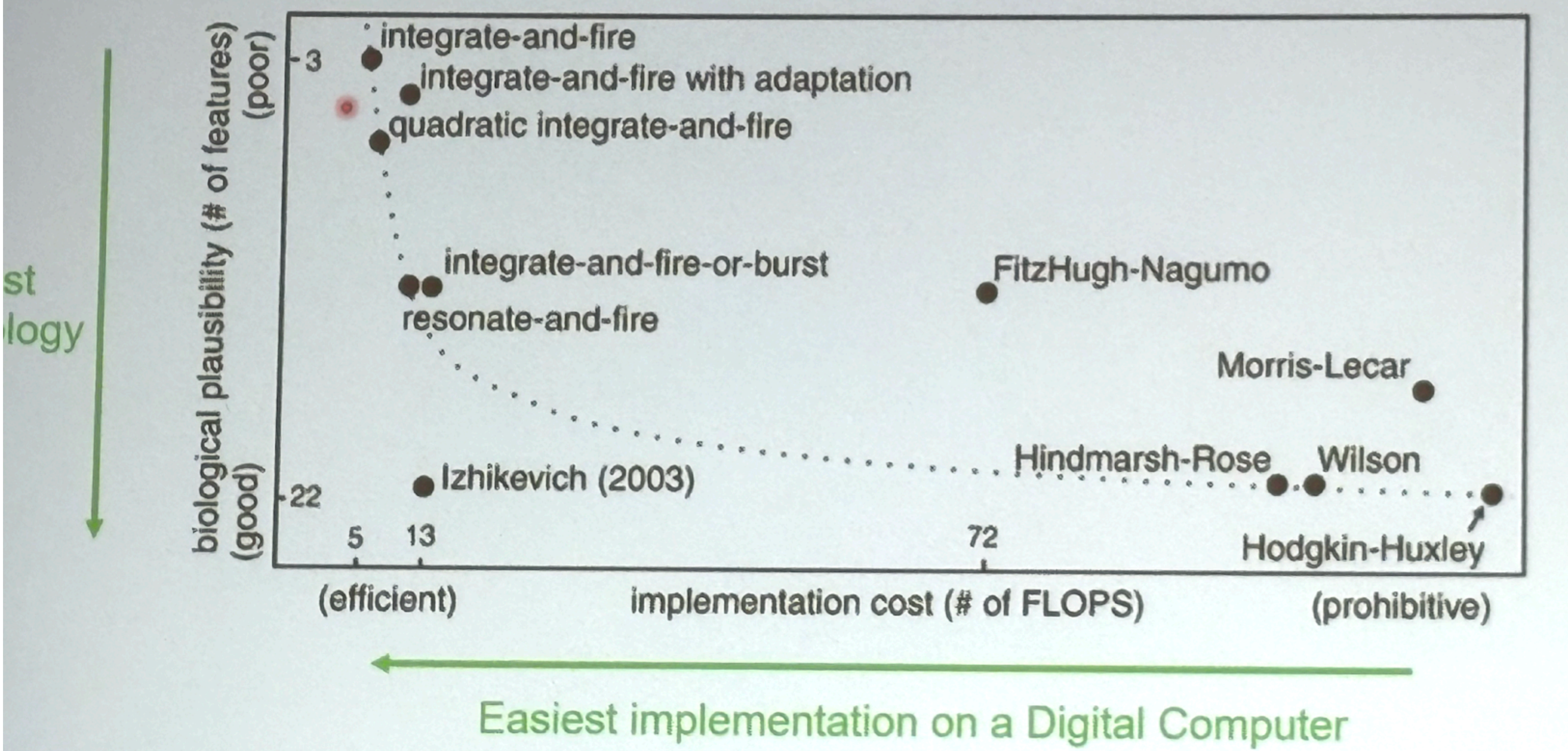
$$\alpha_m = 0{\cdot}1\,(V+25) \Big/ \left(\exp\frac{V+25}{10} - 1\right), \tag{20}$$

$$\beta_m = 4\,\exp\,(V/18), \tag{21}$$

$$\alpha_h = 0{\cdot}07\,\exp\,(V/20), \tag{23}$$

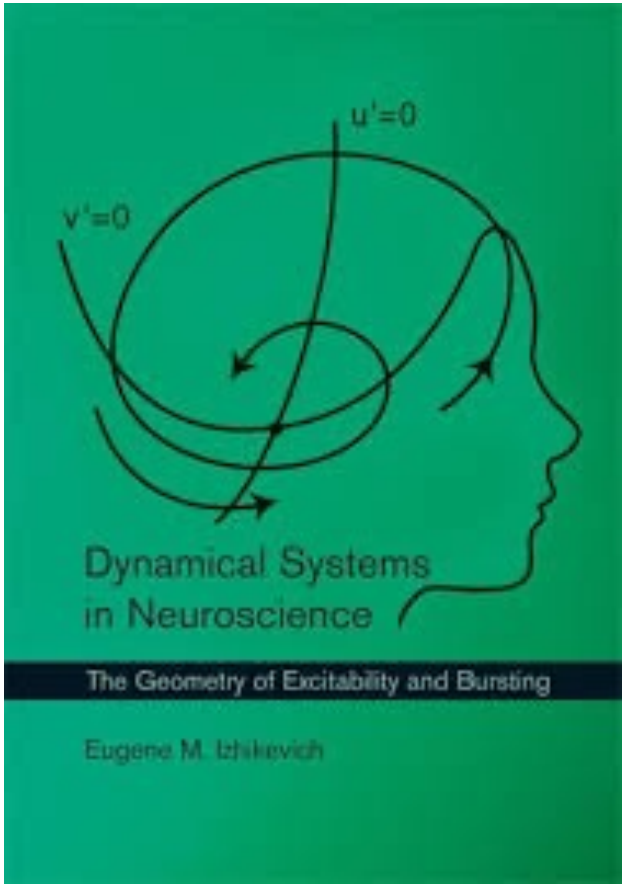
$$\beta_h = 1 \Big/ \left(\exp\frac{V+30}{10} + 1\right). \tag{24}$$

Complex neuron models – trade-off with performance?

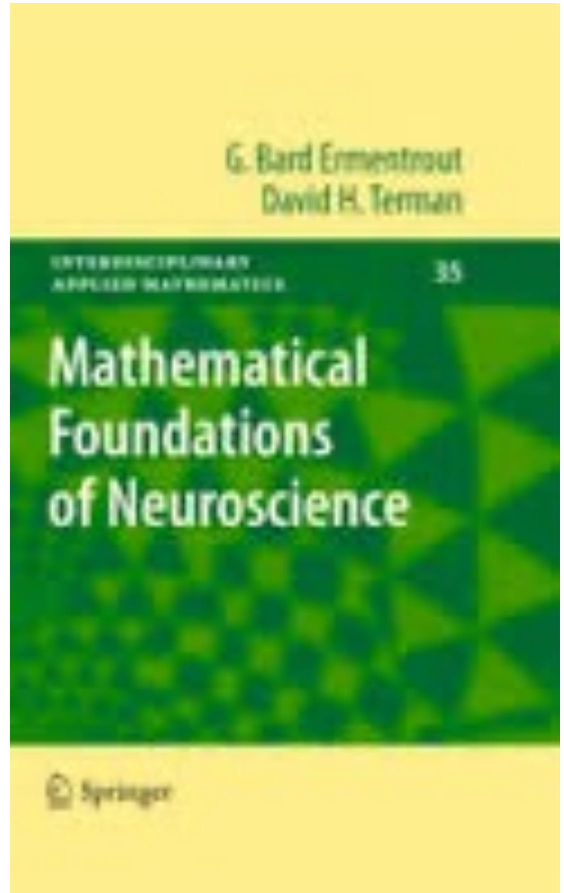


Yamazaki, K.; Vo-Ho, V.-K.; Bulsara, D

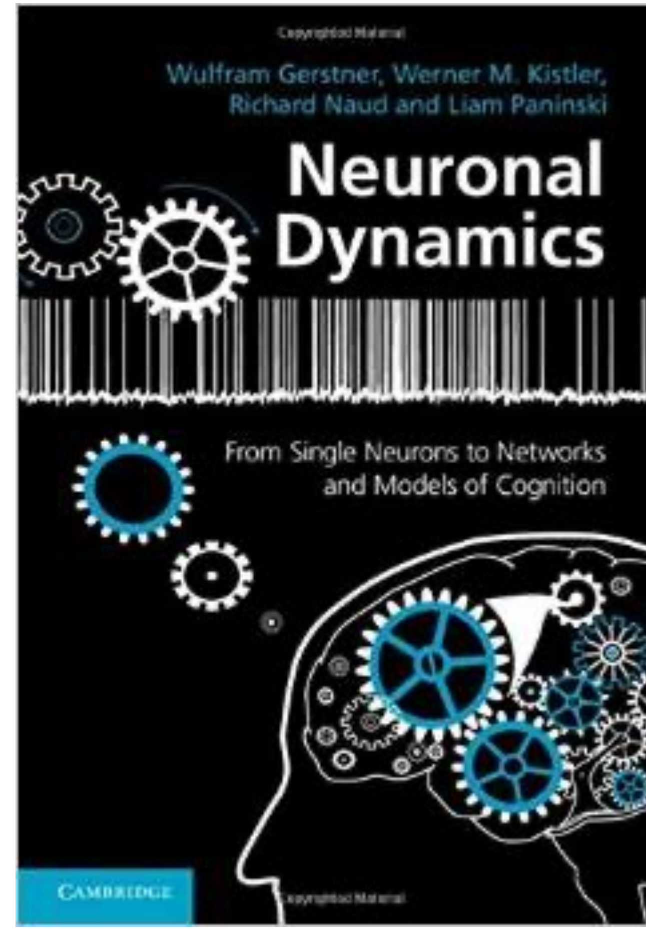
Dynamical systems neuroscience



2007



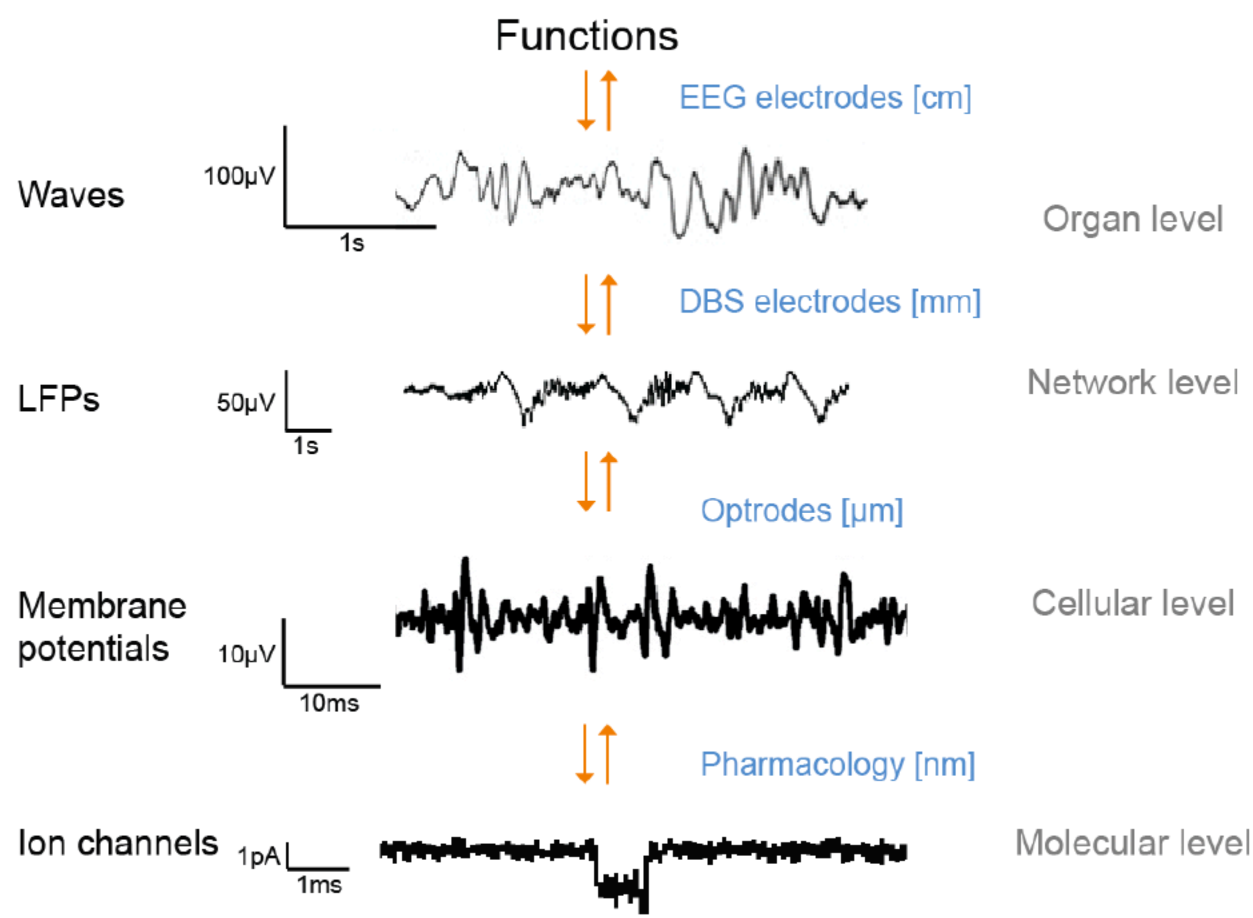
2010



2014

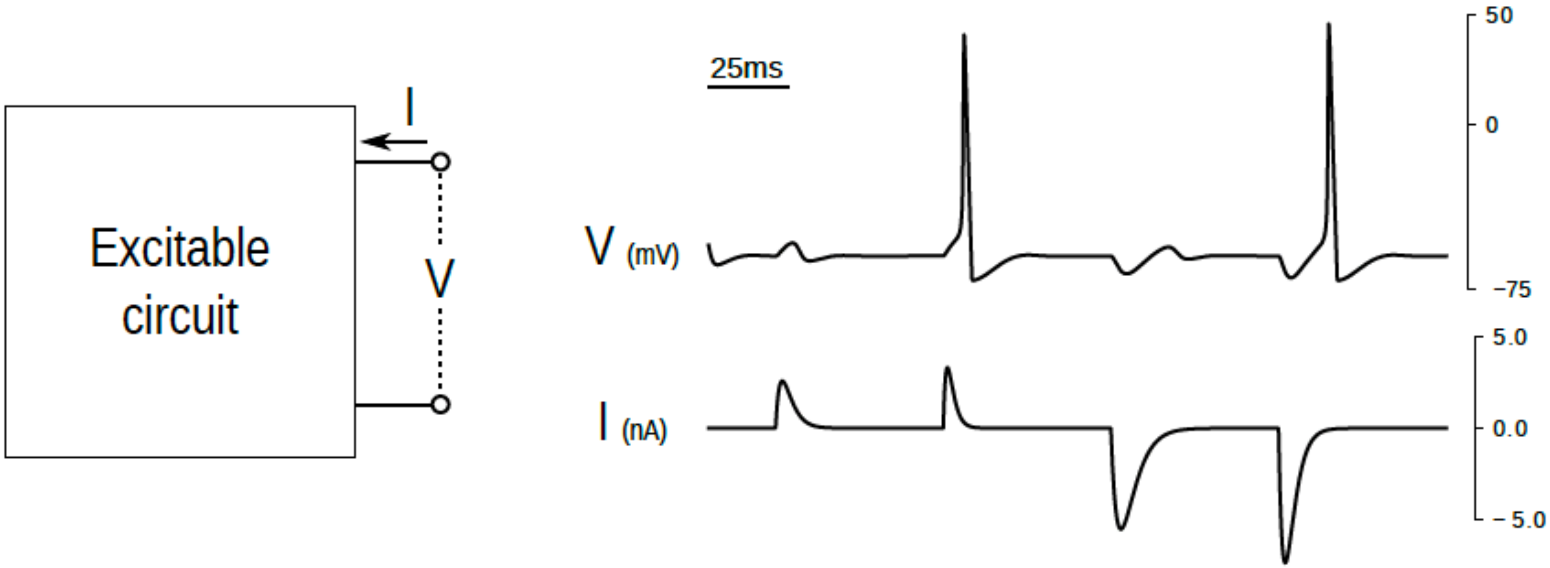
Does mathematical neuroscience acknowledge the physics of neuronal modelling ?

Anzatz 1: neuronal behaviors are multi-scale and event-based



Events should be modelled “*at scale*”

Anzatz 2: events are trajectories of *excitable* models



R. Sepulchre, G. Drion, A. Franci. Excitable Behaviors. In: Tempo R., Yurkovich S., Misra P. (eds) Emerging Applications of Control and Systems Theory. Lecture Notes in Control and Information Sciences - Proceedings. Springer, Cham, 2018.

A tractable theory of excitability should benefit from physical modelling at scale

The reliability experiment

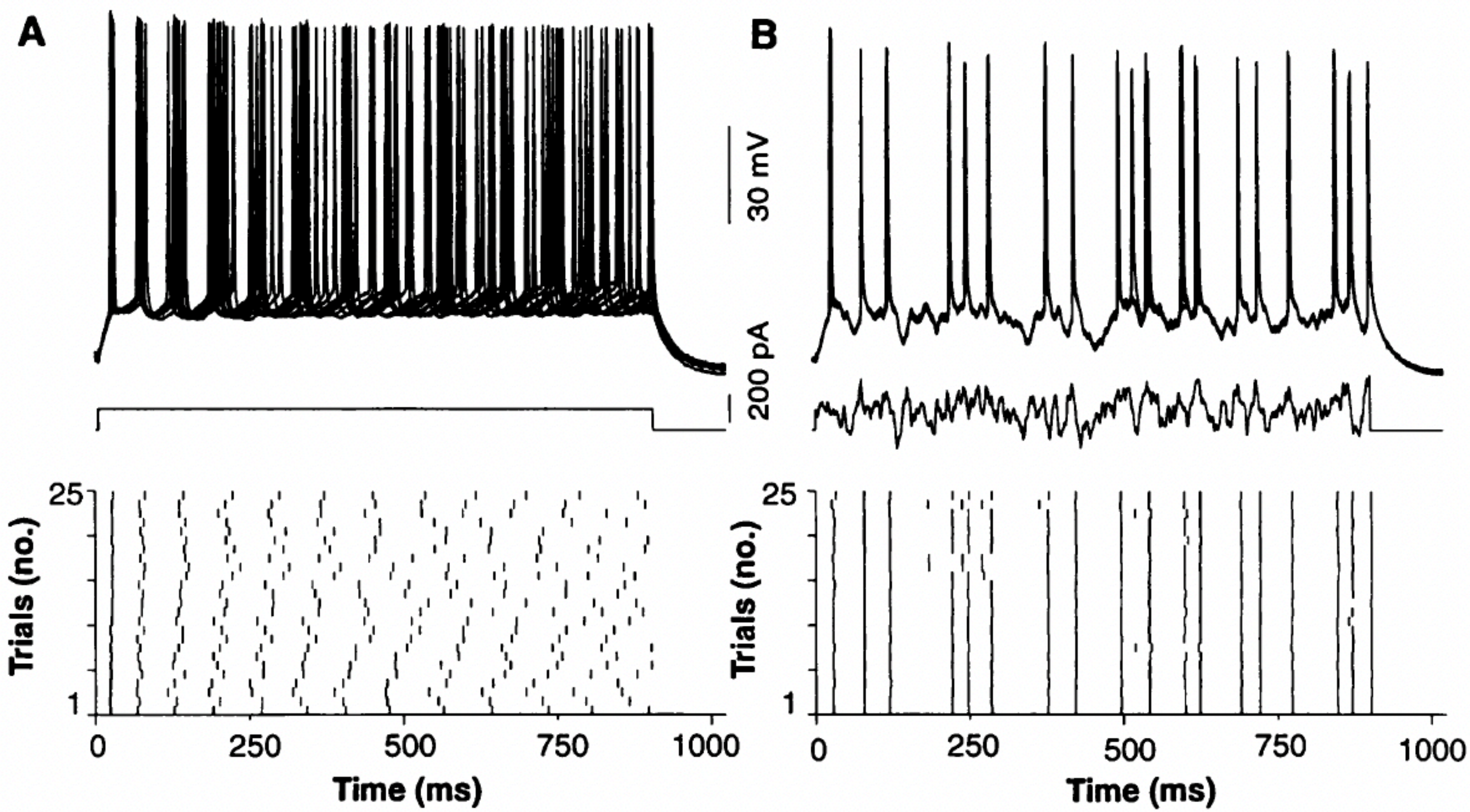
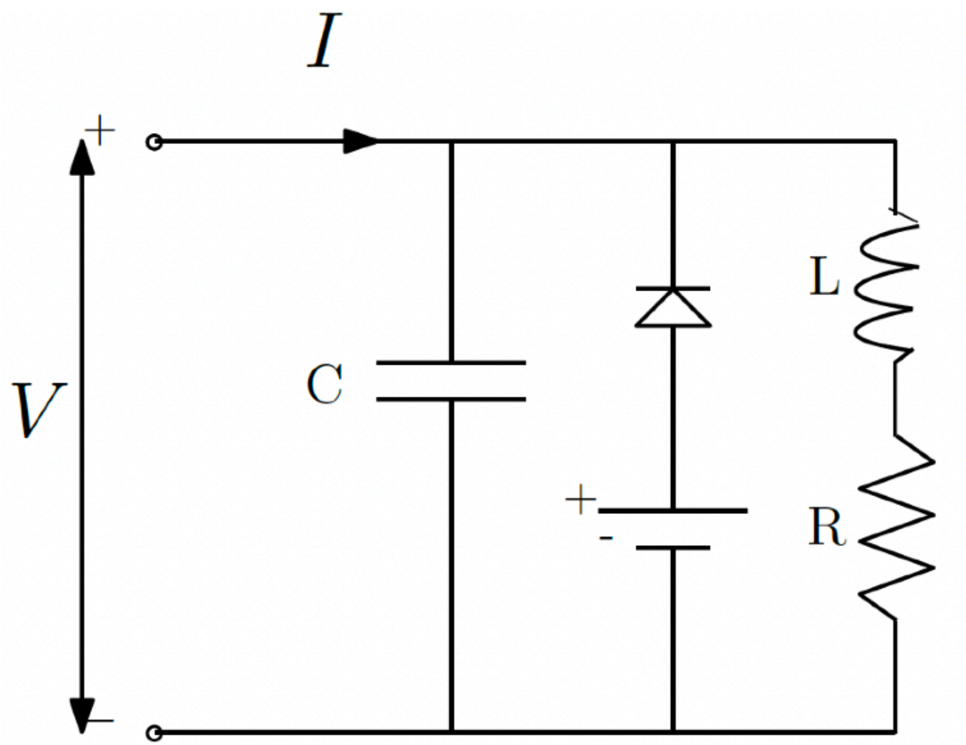


Fig. 1. Reliability of firing patterns of cortical neurons evoked by constant and fluctuating current. (**A**) In this example, a superthreshold dc current pulse (150 pA, 900 ms; middle) evoked trains of action potentials (approximately 14 Hz) in a regular-firing layer-5 neuron. Responses are shown superimposed (first 10 trials, top) and as a raster plot of spike times over spike times (25 consecutive trials, bottom). (**B**) The same cell as in (A) was again stimulated repeatedly, but this time with a fluctuating stimulus [Gaussian white noise, $\mu_s = 150$ pA, $\sigma_s = 100$ pA, $\tau_s = 3$ ms; see (*14*)].

Mainen & Sejnowski, Science, 1995

FitzHugh-Nagumo model



$$
\begin{aligned}
C\dot{V} &= kV - \frac{V^3}{3} - I_L + I_{ext} \\
L\dot{I}_L &= -I_L + RV
\end{aligned}
$$

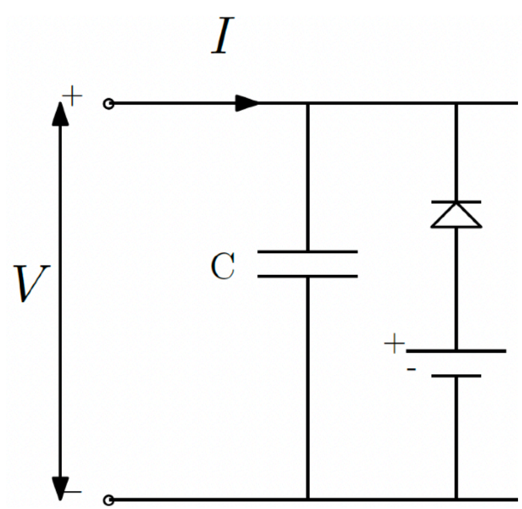
A circuit that reproduces the mechanism of nerve impulse:

R. FitzHugh, "Impulses and physiological states in theoretical models of nerve membrane," Biophysical journal, vol. 1, no. 6, p. 445, 1961.

J. Nagumo, S. Arimoto, and S. Yoshizawa, "An active pulse transmission line simulating nerve axon," Proceedings of the IRE, vol. 50, no. 10, pp. 2061–2070, 1962.

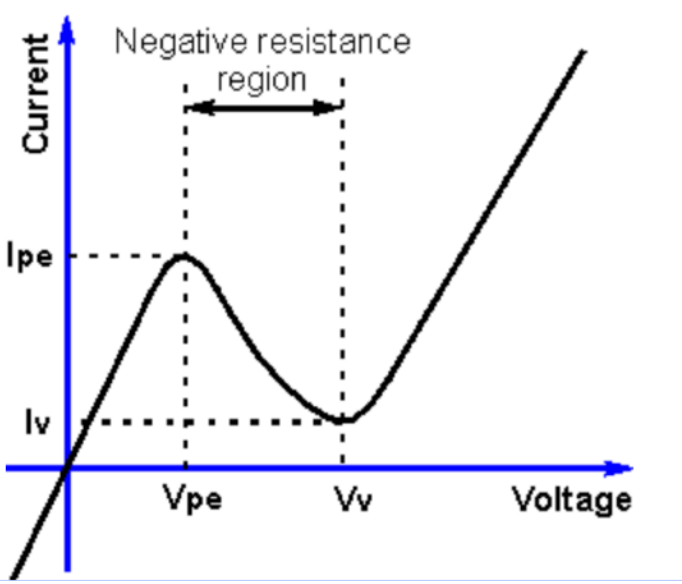
Referred to as "Bonhoeffer-van der Pol model" by FitzHugh after Van der Pol (1926).

The memory of FN circuit

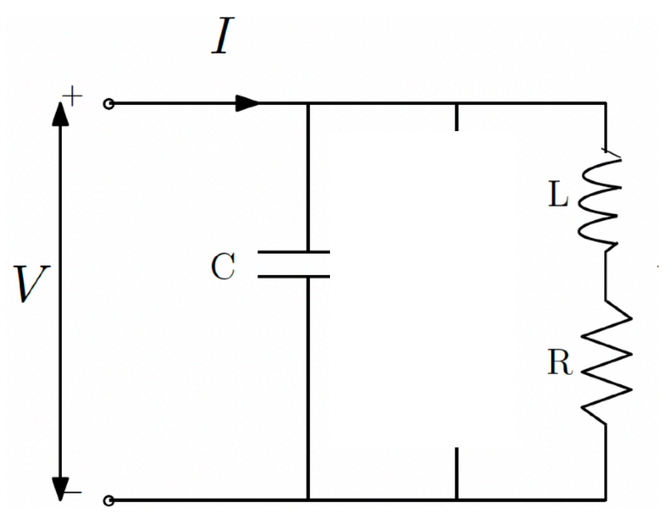


$$C\dot{V} = kV - \frac{V^3}{3} + I_{ext}$$

For a range of constant current,
bistable memory made of a capacitor (physical storage)
and a negative resistance device



The fading memory of FN circuit



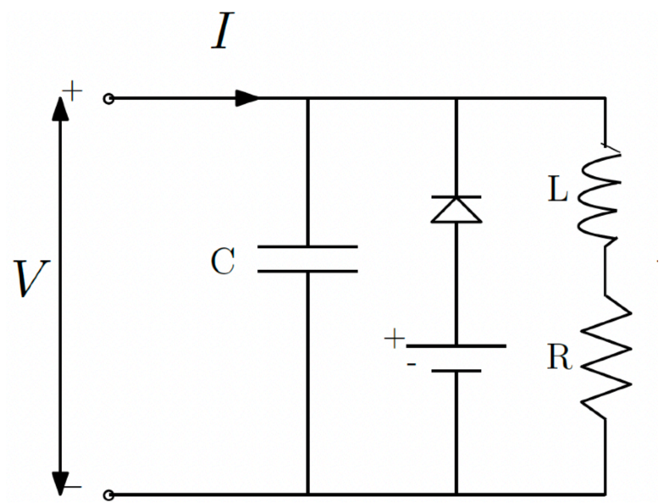
$$C\dot{V} = -I_L + I_{ext}$$
$$L\dot{I}_L = -I_L + RV$$

A RLC circuit has fading memory:
the effect of a current impulse fades out with time.

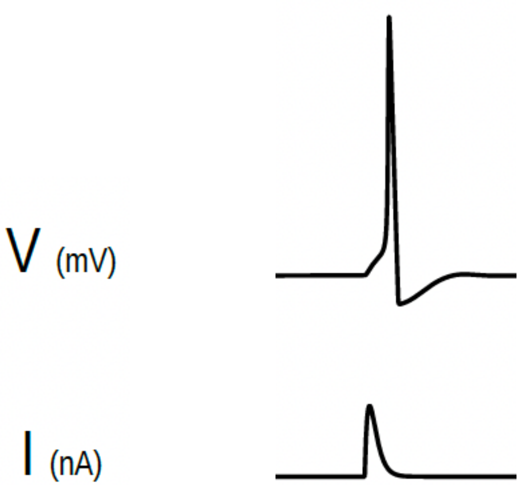
The elements R, L, and C, shape the fading memory

If the ration C/L is small, a current impulse charges the capacitor almost instantaneously, and the time constant L/R dictates the fading memory.

Thresholds result from mixing memory and fading memory



$$
\begin{aligned}
C\dot{V} &= kV - \frac{V^3}{3} - I_L + I_{ext} \\
L\dot{I}_L &= -I_L + RV
\end{aligned}
$$



Transient switch =
Memory at ‘fine’ scale
Fading memory at ‘coarse’ scale

What is ‘scale’ ? A mixture of amplitude and time …

Motivation: the complexity of neuronal modelling

- Neuronal behaviors are excitable, multi-scale, and event-based
- Methods from dynamical system theory (fast/slow analysis, mean-field modelling, …) tame the complexity at the expense of physical modelling
- Physical modelling at scale: can we simplify the mathematical law of the behavior *before* rather than *after* the modelling step ?

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Signal modelling at scale: wavelets

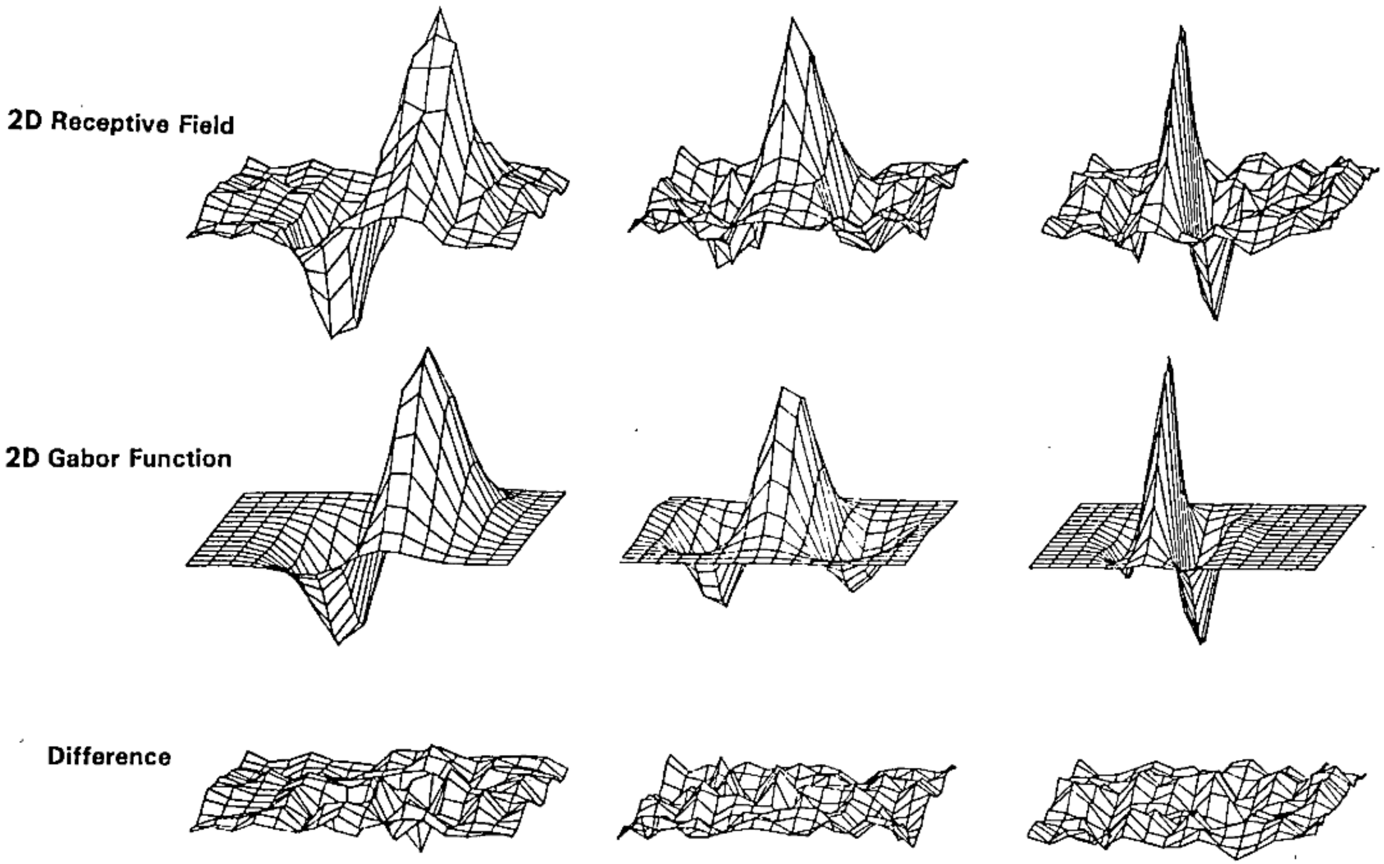

Fig. 3. Illustration of experimentally measured 2D receptive-field profiles of three simple cells in cat striate cortex (top row) obtained in the laboratory of L. A. Palmer and J. P. Jones (University of Pennsylvania Medical School). Each plot shows the excitatory or inhibitory effect of a small flashing light or dark spot on the firing rate of the cell, as a function of the (x, y) location of the stimulus, computed by reverse correlation of the 2D stimulus sequence with the neural-response sequence. The second row shows the best-fitting 2D Gabor function for each cell's receptive-field profile, based on Eqs. (3) with the parameters fitted by least squares. The third row shows the residual error between the measured response profile of each cell and Eqs. (3). In formal statistical tests, the residuals were indistinguishable from random error for 33 of the 36 simple cells tested. (From Ref. 28.)

Signal modelling at scale: wavelets

A *mother* signal $V(t,x)$

A scaled signal $V_{\mathrm{scaled}}(t,x) = \frac{V(\frac{t-a}{\tau}, \frac{x-b}{\sigma})}{s}$

Amplitude scaling : s Temporal scaling : τ Spatial scaling : σ

Temporal shifting : a Spatial shifting : b

- The mother signal is localized in time, space, and amplitude i.e. an *event*
- The complexity arises from a mixture of multi-scale events
- Taming the complexity comes from acknowledging the event-based nature of the complex signal *before* the analysis

Behavioral modeling at scale

A *mother* behavior : relationship between $V(t,x)$ and $I(t,x)$

A *scaled* behavior : relationship between $V_{\text{scaled}}(t,x)$

… and a corresponding scaled current

Invariance with respect to time and space shifting is a consequence of time- and space- invariance.

Invariance with respect to amplitude scaling is a consequence of linearity.

Invariance is too much to ask for. How to define a behavior at scale ?

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Circuit modelling at scale

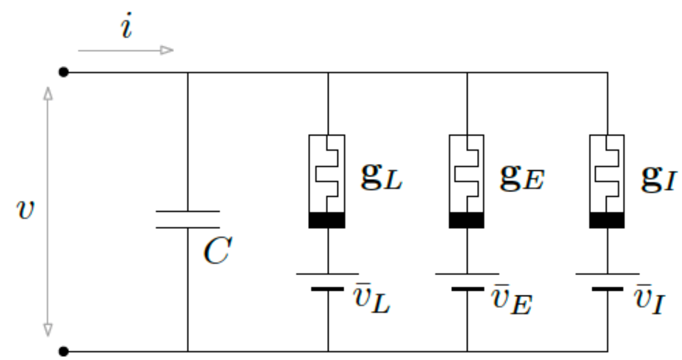


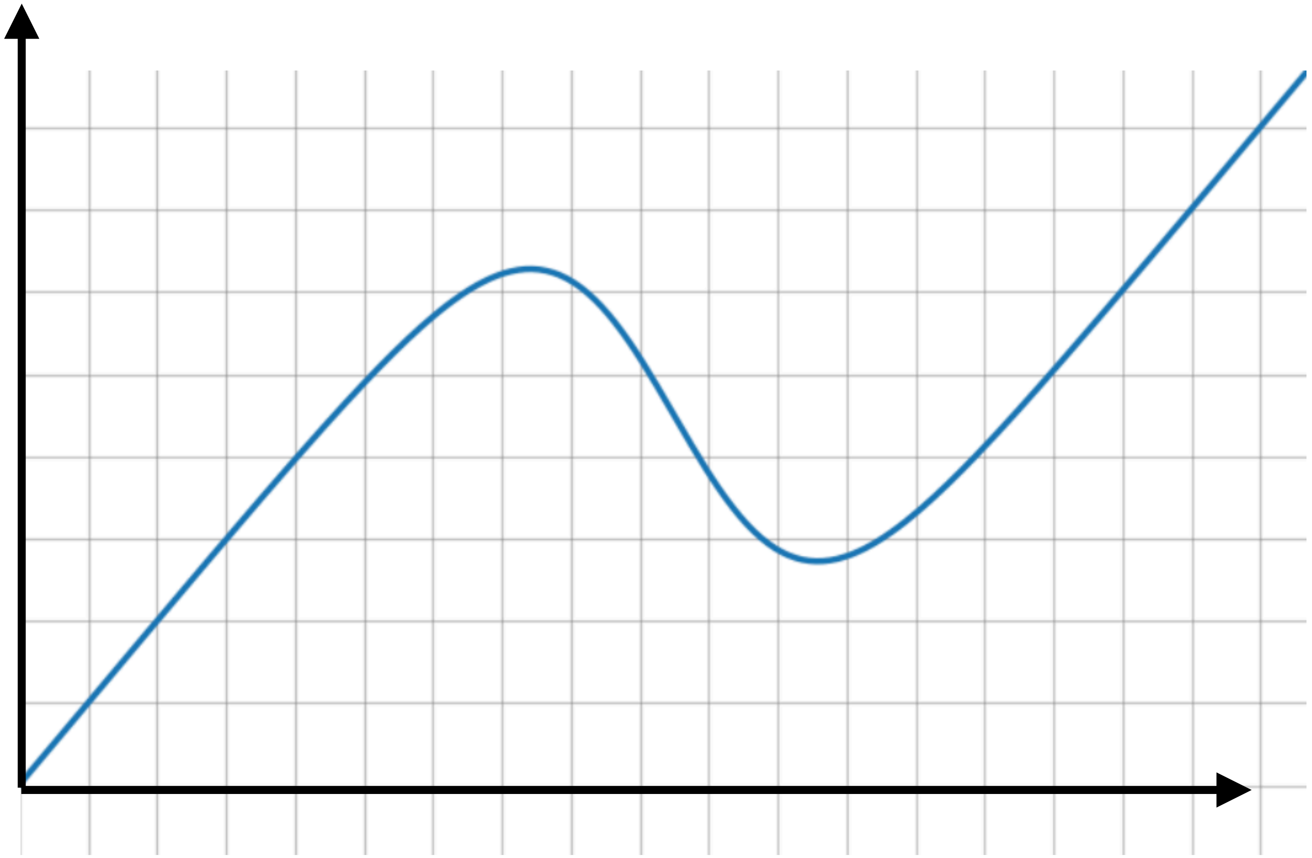
Fig. 2. Conductance-based neuron equivalent circuit.

- Retain the circuit architecture: power-preserving interconnection of circuit elements

- Retain the nature of circuit elements: either store energy or dissipate energy

- Only the constitutive law of elements should be *at scale*

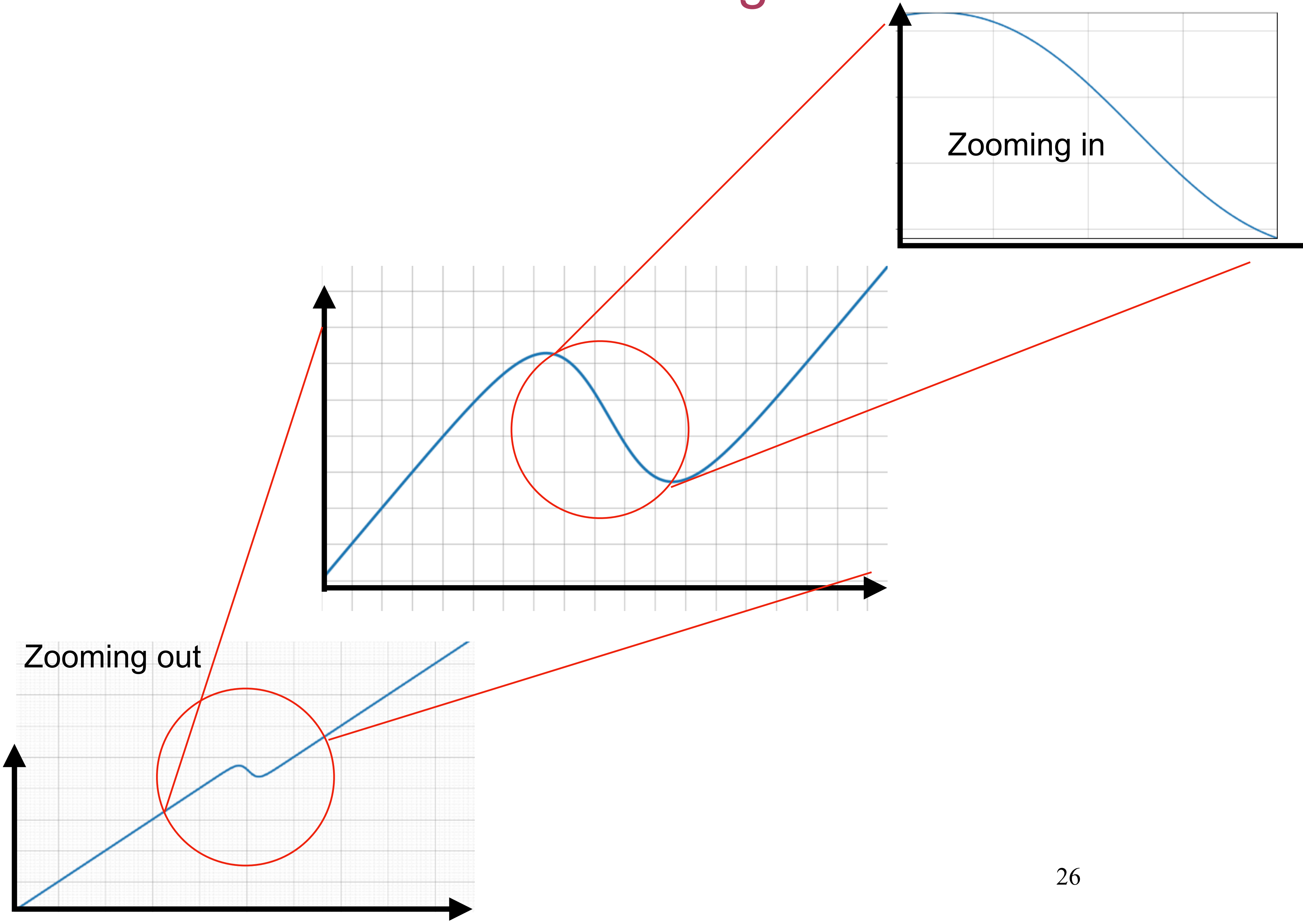
Resistive modelling at scale



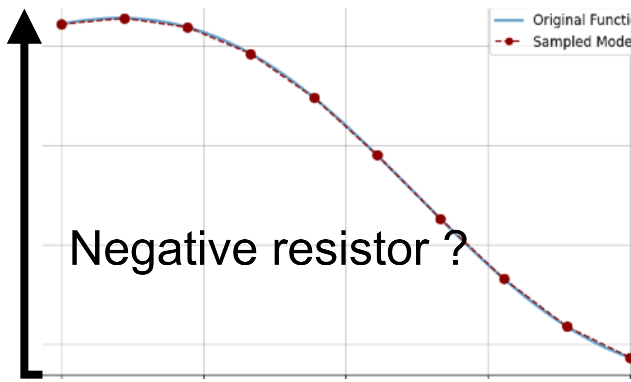
Negative resistance is an example of a property at scale

Key behavioral property: invertible vs non-invertible; memory vs fading memory

Resistive modelling at scale

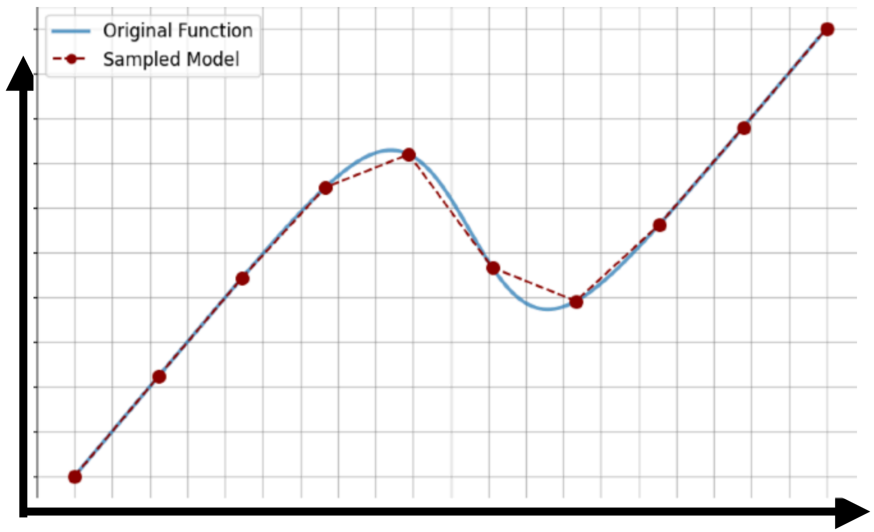


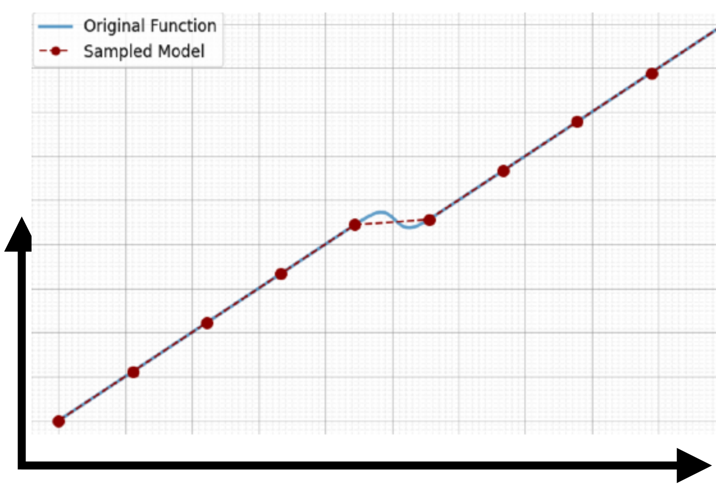
Constitutive law at scale



Negative resistor ?

Zooming in





Monotone resistor ?

Circuit modelling at scale

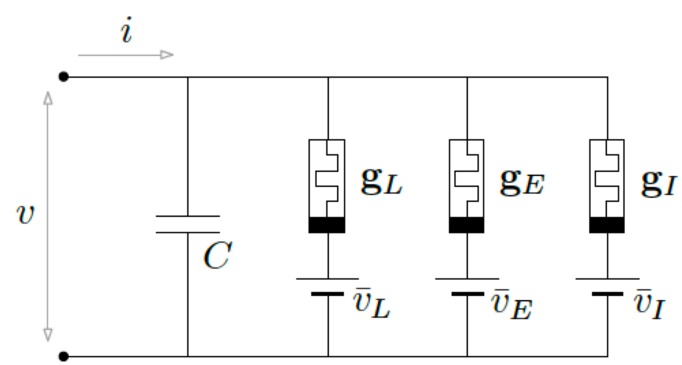


Fig. 2. Conductance-based neuron equivalent circuit.

- How can we relate the scale of the elements to the scale of the behavior ?
- How can we guarantee that *scale* is commensurate with *complexity* ?

Proposal: energy modelling at scale

1. Exploit the **energy characterization** of each element.

For instance: a resistor is characterized by its "content" or "co-content".

2. Use the bumpiness of the **energy landscape** (e.g. amount of critical points) as a measure of complexity.

For instance: the content of a monotone resistor is convex. The content of a negative resistor is a double well potential.

3. "**Coarsening**" should correspond to "**simplifying**". The energy landscape should be a *variation diminishing* function of scale.

For instance: fine scale modelling should acknowledge the double well. Coarse scale modelling could approximate the double well by a convex landscape

Energy modelling of neuronal circuits

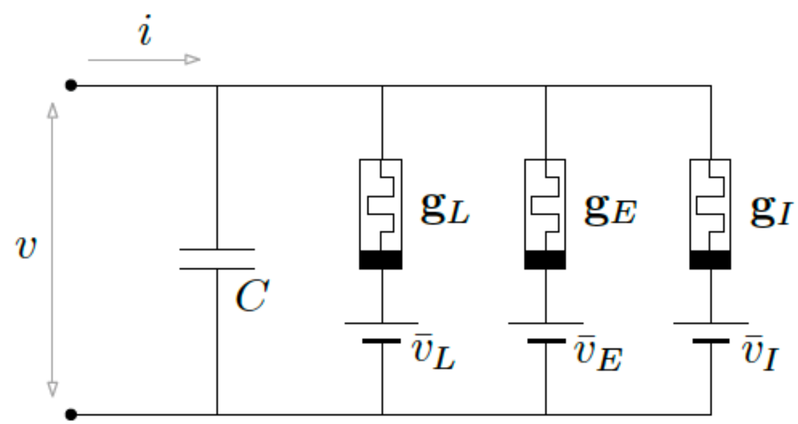


Fig. 2. Conductance-based neuron equivalent circuit.

One *storage* element with capacitive energy $S(v) = \frac{1}{2}Cv^2$

An energy balance:

$$\underbrace{\dot{S} - vi}_{\text{dissipation}} = v\left(\sum_k g_k(v - \bar{v}_k)\right) = \underbrace{g_{tot}v^2}_{\text{Total power dissipated in memristive elements}} - \underbrace{v\sum_k g_k\bar{v}_k}_{\text{Total power extracted from internal batteries}}$$

The energy landscape is determined by the total dissipated energy

Energy-based modelling of memristive elements

(from the previous talk)

Memristor

$$\mathbf{i}(t) = \mathbf{g}(\mathbf{v})(t)\,\mathbf{v}(t),$$

Inner products

$$\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = \int_{-\infty}^{T} \mathbf{v}_1(t)\mathbf{v}_2(t)\mathrm{d}t \qquad \langle \mathbf{v}_1, \mathbf{v}_2 \rangle_{\frac{1}{\mathbf{g}}} = \int_{-\infty}^{T} \frac{\mathbf{v}_1(t)\mathbf{v}_2(t)}{\mathbf{g}(\mathbf{v})(t)}\mathrm{d}t$$

1/g - dissipated energy

$$\mathcal{D}_{\frac{1}{\mathbf{g}}} = \langle \mathbf{v}, \mathbf{v} \rangle$$

1/g co-content

$$\mathcal{E}_{\frac{1}{\mathbf{g}}} = \frac{1}{2}\langle \mathbf{v}, \mathbf{v} \rangle$$

The 1/g co-content <u>determines</u> the resistive relationship

$$\left\langle \mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}), \mathbf{v} \right\rangle_{\frac{1}{\mathbf{g}}} = \mathbf{D}_{\mathbf{v}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) = \langle \mathbf{v}, \mathbf{v} \rangle\,, \qquad \mathbf{i} = \mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) = \mathbf{g}(\mathbf{v})(\cdot)\mathbf{v}(\cdot)$$

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Summary of the question

Neuronal modelling at scale = memristive modelling at scale

Memristive modelling = memductance modelling = functional from past voltage

Modelling at scale = define a scaled memductance from the mother memductance = functional from the past scaled voltage

Wanted: the scaled memductance is a variation diminishing function of the scale.

Resistive modelling at scale

The mother conductor is $$i = g(v)v$$

We look for a scaled conductor $$i_s = g_s(v_s)v_s$$

where $$v_s = \frac{v}{s}, \; s > 0$$

so that

(i) the *scaled* conductance can be evaluated from the *mother* conductance

(ii) the *scaled* conductance is a variation diminishing function of the scale.

Generalised linear modelling

Let $g(v) = g_0\, m(v)$

and $\log m(v) = \int_0^\infty M(a) \log(1 + \exp(av)) da$

Think of $M(a)$ as a “scale” transform of m

THEN:

- Rescaling v amounts to rescale a, that is “stretching” the scale plot
- Coarsening g amounts to “band passing” M

Key aspects: “linear” sampling and approximation theory in the log space
+ “exponential” basis functions

Temporal modelling at scale

Amplitude scaling of g(v) is analog to *time* scaling of $i(t) = v(t) * g(t)$

Assuming *real* poles and zeros, we have $G(s) = G_0 \frac{\Pi_{i=1}^{M}(1+\tau_i s)^{k_i}}{\Pi_{j=1}^{N}(1+\tau_j s)^{l_j}}$

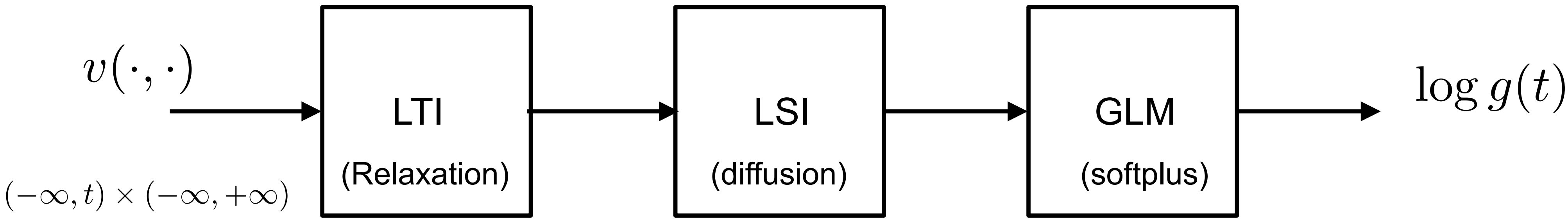
$$\log \mid G(j\omega) \mid = \log G_0 + \frac{1}{2}\sum_i k_i \log(1+\tau_i^2\omega^2) - \sum_j l_j \log(1+\tau_j^2\omega^2)$$

In log coordinates $a = \log\omega \quad a_i = \log\tau_i$

$$\log \mid G \mid = \log G_0 + \frac{1}{2}(\sum_i k_i \log(1+\exp(a+a_i)) - \sum_j l_j \log(1+\exp(a+a_j)))$$

- Rescaling time amounts to rescale frequency, that is “shifting” the Bode plot
- Coarsening g amounts to “band passing” the Bode plot

Memristive modelling at scale



- Rescaling time/space/amplitude amounts to rescale Bode and scale plots
- Coarsening amounts to band-passing (temporal frequency, spatial frequency, and amplitude scale)

Have we answered the question ?

Neuronal modelling at scale = memristive modelling at scale

The memductance is a functional from past voltage

Scaling the memductance amounts to shift/stretch in the frequency/scale domain

The scaled memductance is a variation diminishing function of the scale.

Contents

Physical modelling at scale

- Constrain the modelling with physical principles: power-preserving interconnections of basic elements characterized by their energy.

- Model the constitutive law of elements *at scale*

- Make energy a variation diminishing function of scale through exponential linear modelling

Perspectives

- A compelling, multi-scale characterization of excitability: memory at fine scale, fading memory at coarse scale.
- Design: energy-shaping = memory-shaping can be done at any scale
- A system theoretic alternative to mean-field modelling.

Research directions

- Modelling at scale can be regarded as a distinction between "*physics*" (the mother behavior) and "*perception*" (the scaled and band-passed behavior).
- Neuronal circuits = memristive Hopfield networks = multi-resolution associative memories
- Designing a neuronal circuit is shaping the memory of a machine

Connections deserving further attention

- Scale-space theory in computer vision
- Dynamic input conductances in neurophysiology
- Statistical modelling with GLMs (logistic regression, …)
- Scaled Riemannian metrics and affine-invariant geometry
- Scaled exponential kernels for analysis and synthesis
- Neuroscience of perception and memory

Neuromorphic vs Ising/quantum

- Classical LTI dissipativity theory specialises “reciprocal passive” systems (RLC) into “relaxation” systems (RC) and “lossless” systems (LC).
- Neuronal circuits can be thought as “memristive” generalisations of relaxation systems (with batteries).
- What about a “memductive” generalization of lossless systems?
- What does mean shaping the storage of such systems “at scale” ?

Conclusion

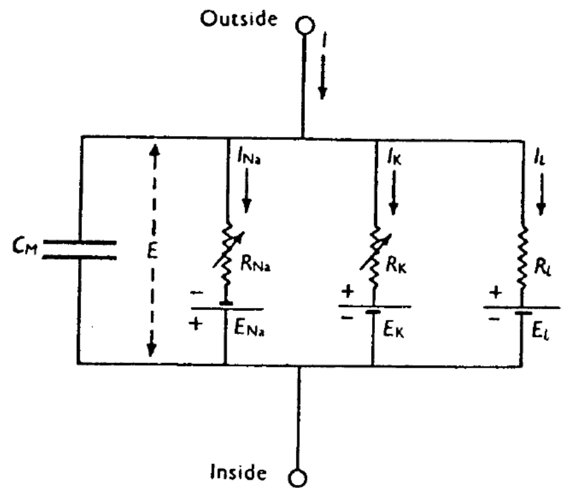
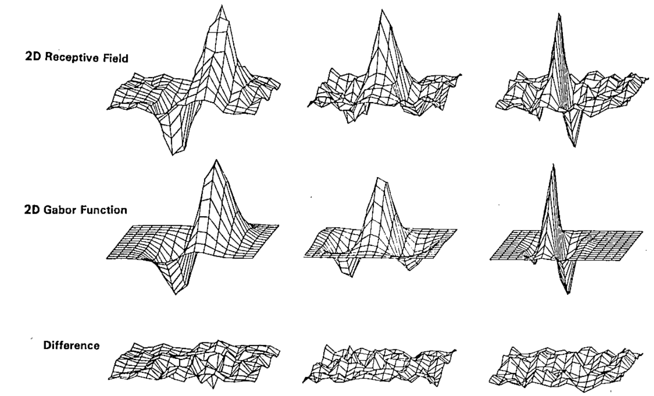


Fig. 1. Electrical circuit representing membrane. $R_{Na} = 1/g_{Na}$; $R_K = 1/g_K$; $R_l = 1/\bar{g}_l$. R_{Na} and R_K vary with time and membrane potential; the other components are constant.



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- Physical modelling is a precious source of algorithmic tractability.
- Multi-scale *modelling* can facilitate multi-scale *analysis* and *synthesis*
- Scale-space theory can be regarded as a physical theory of wavelets.
- Work in progress: a physical theory of *switchlets*.