UNIVERSITY OF CAMBRIDGE

Gradient modeling of memristive circuits

Fulvio Forni, Rodolphe Sepulchre

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Gradient modeling of memristive circuits

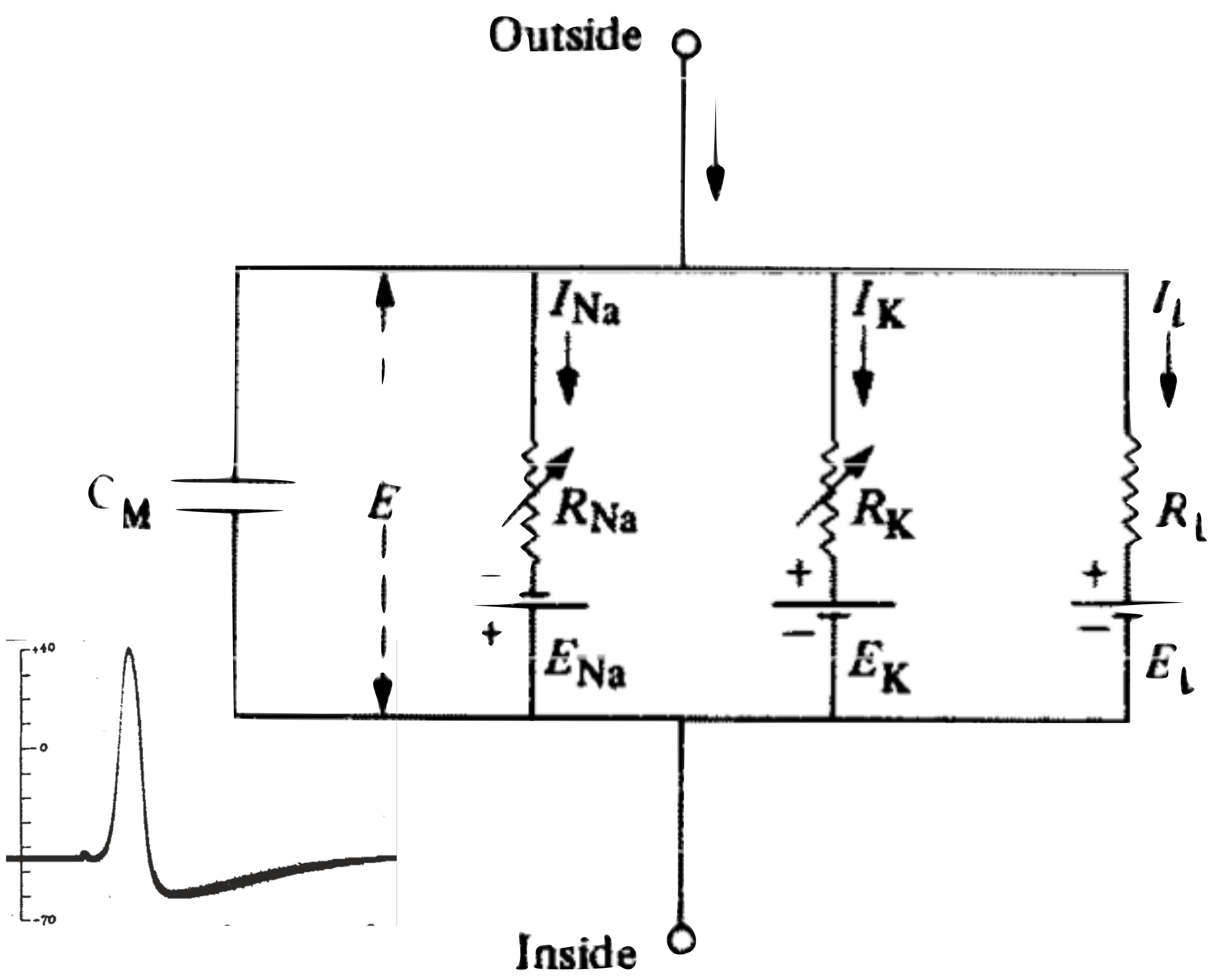
Part 1: Motivation

Part 2: Gradient resistive and memristive elements

Part 3: linear RC and mem-RC circuits

Part 4: On memcapacitance and excitable robotics…

Motivation 1: **the Hodgkin-Huxley's neuron (1952)**



$$I = C_M \frac{dV}{dt} + I_{Na} + I_K + I_l.$$

$$I_{Na} = g_{Na}(V - V_{Na}),$$

$$I_K = g_K(V - V_K),$$

$$I_l = \bar{g}_l(V - V_l),$$

A.L. Hodgkin, A.F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve
Bulletting of Mathematical Biology Vol. 52, No. 1/2, pp. 25-71. 1990

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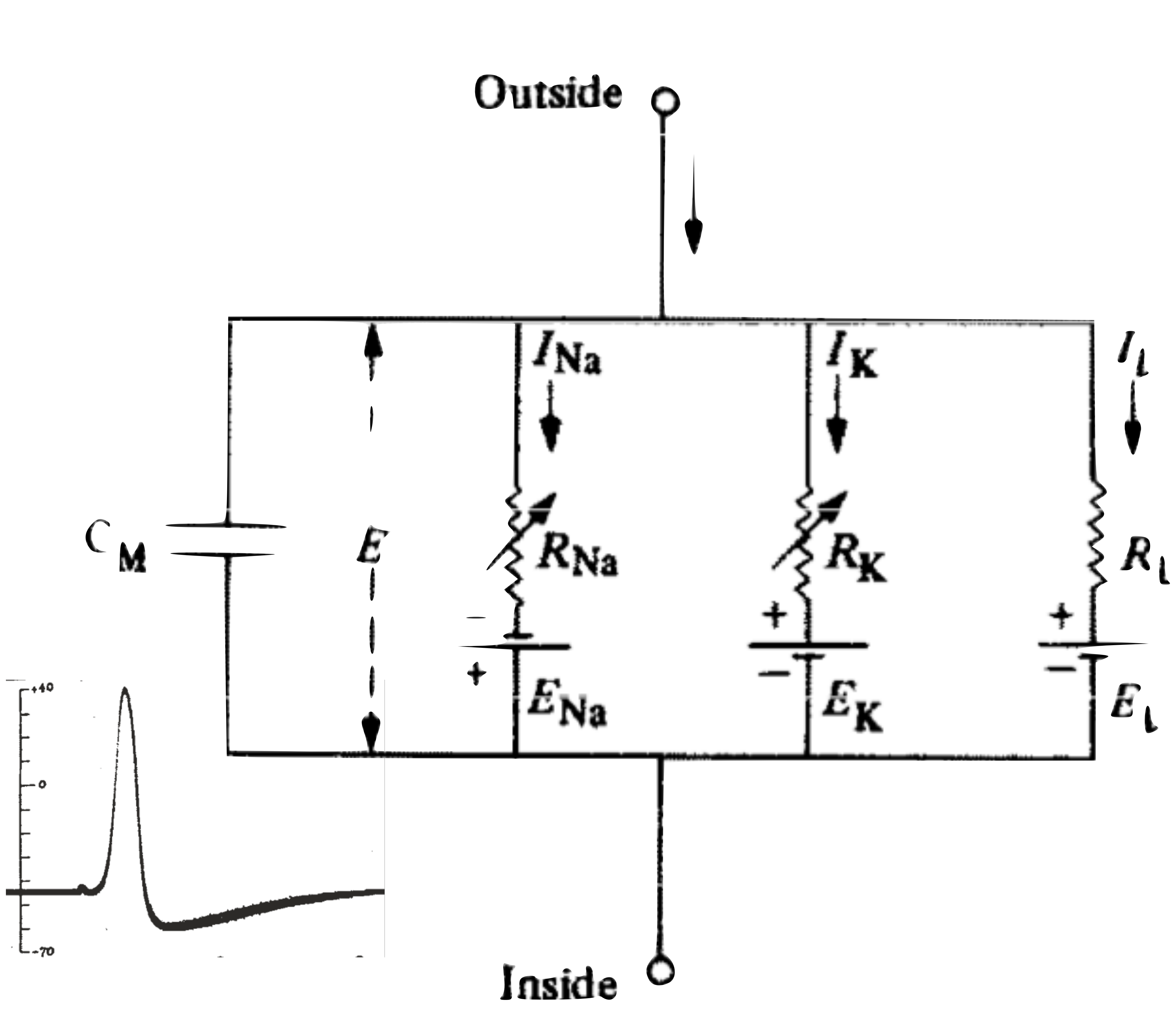

$$I = C_M \frac{dV}{dt} + g_K(V - V_K) + g_{Na}(V - V_{Na}) + \bar{g}_l(V - V_l),$$

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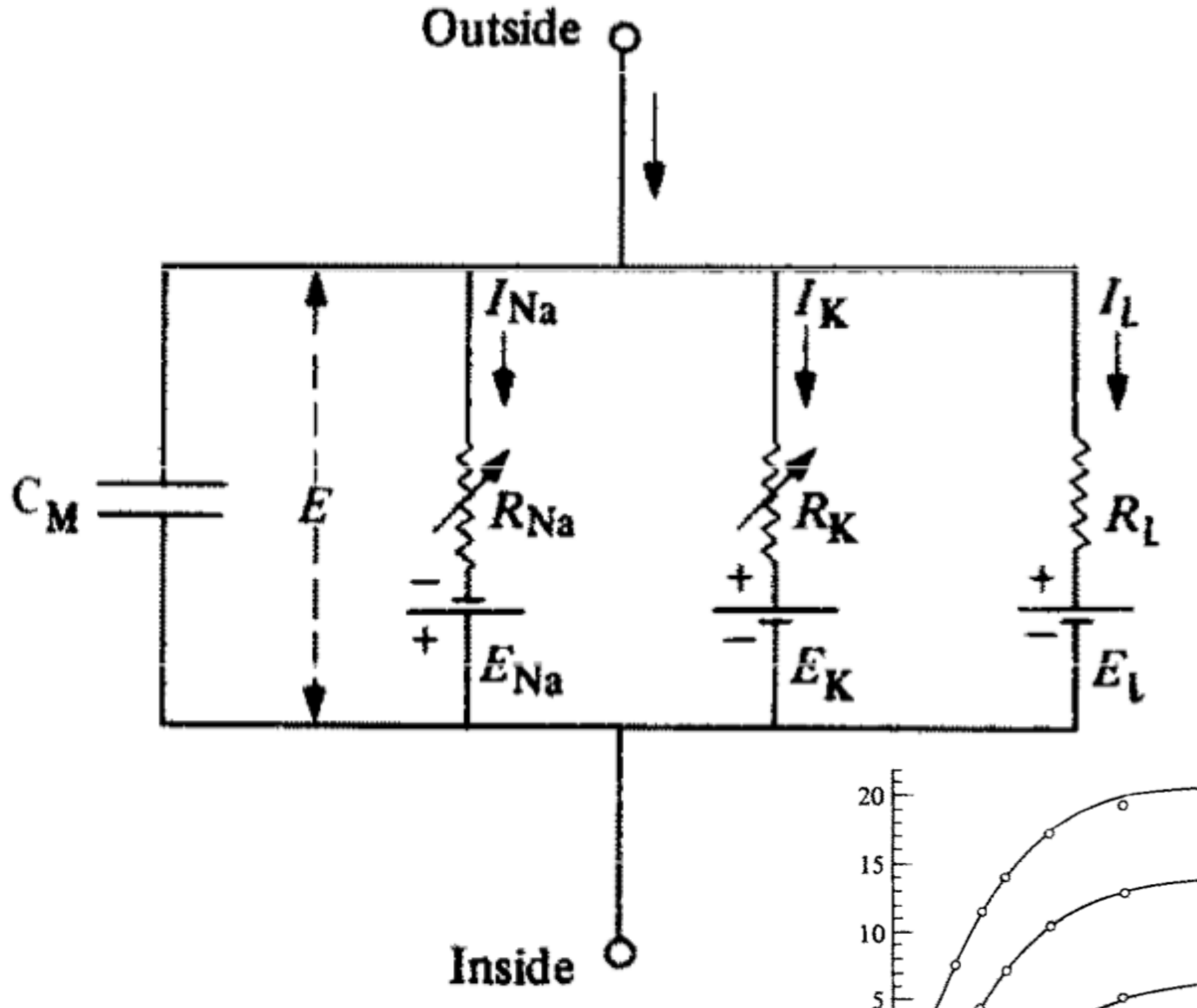
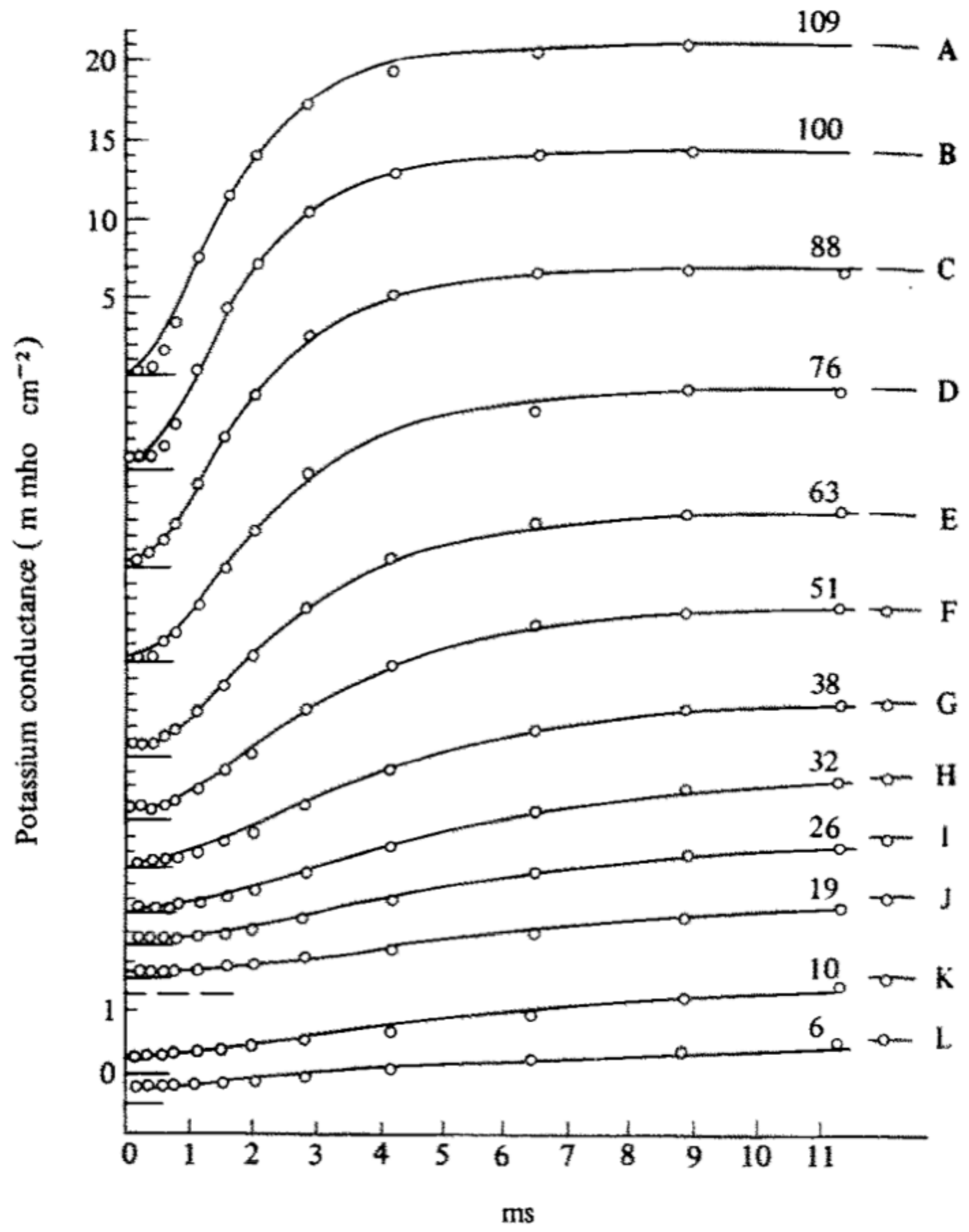




$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m3h(V - V_{Na}) + \bar{g}_l (V - V_l),$$

$$dh/dt = \alpha_h (1 - h) - \beta_h h,$$

$$dn/dt = \alpha_n (1 - n) - \beta_n n,$$

$$dm/dt = \alpha_m (1 - m) - \beta_m m,$$

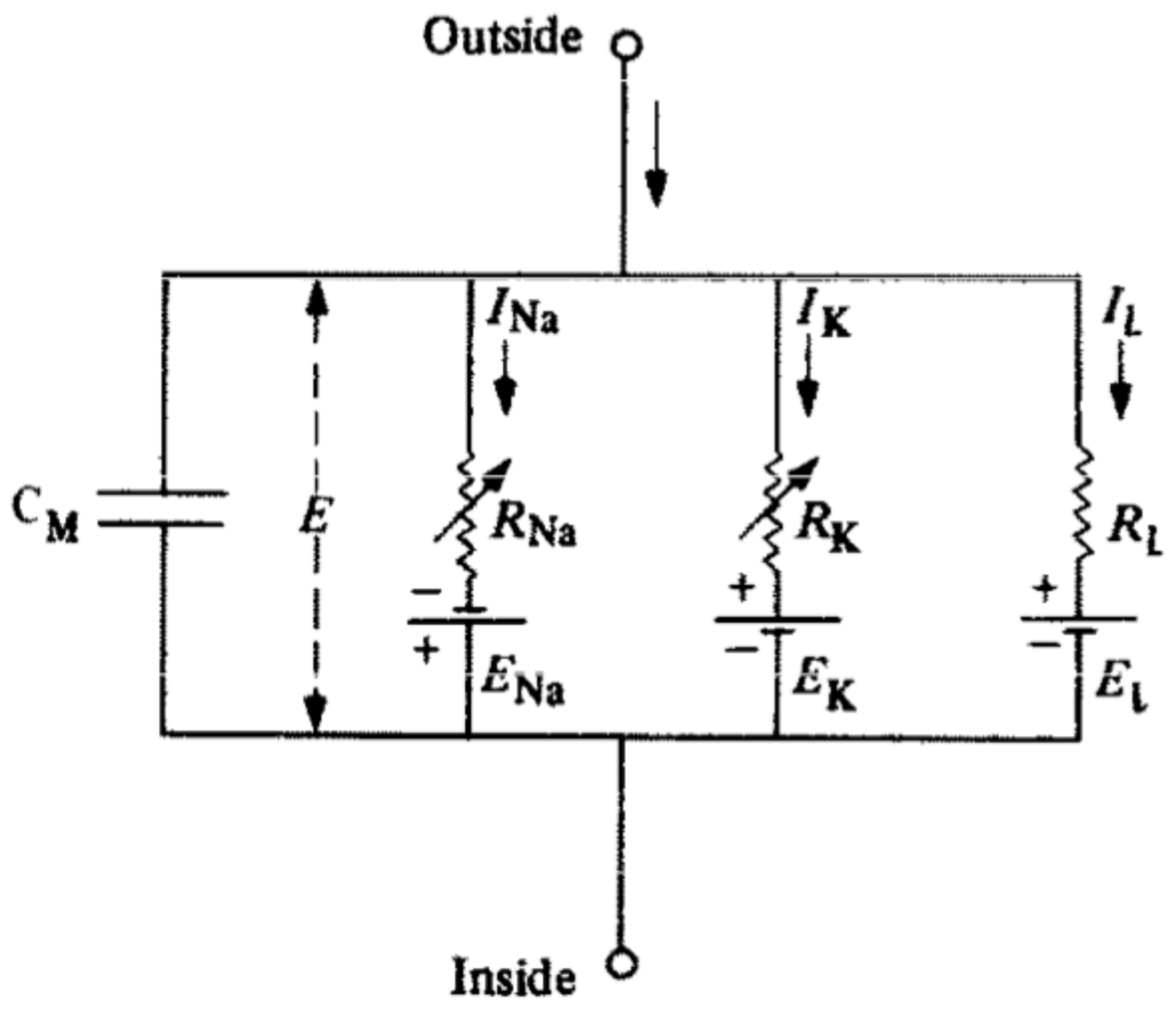
$$\alpha_n = 0.01(V + 10) \Big/ \left(\exp \frac{V + 10}{10} - 1 \right), \qquad \beta_n = 0.125 \exp(V/80),$$

$$\alpha_h = 0.07 \exp(V/20), \qquad \beta_h = 1 \Big/ \left(\exp \frac{V + 30}{10} + 1 \right).$$

$$\alpha_m = 0.1(V + 25) \Big/ \left(\exp \frac{V + 25}{10} - 1 \right), \qquad \beta_m = 4 \exp(V/18),$$

A.L. Hodgkin, A.F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve
Bulletting of Mathematical Biology Vol. 52, No. 1/2, pp. 25-71. 1990

Motivation 2: **Chua's memristive system (1976)**



$$\dot{x} = f(x, i, t)$$
$$v = R(x, i, t)i$$

If $R(x, i) \geqslant 0$, then

$$\boxed{\begin{aligned} \dot{x} &= f(x, u, t) \\ y &= g(x, u, t)u \end{aligned}}$$

Passive

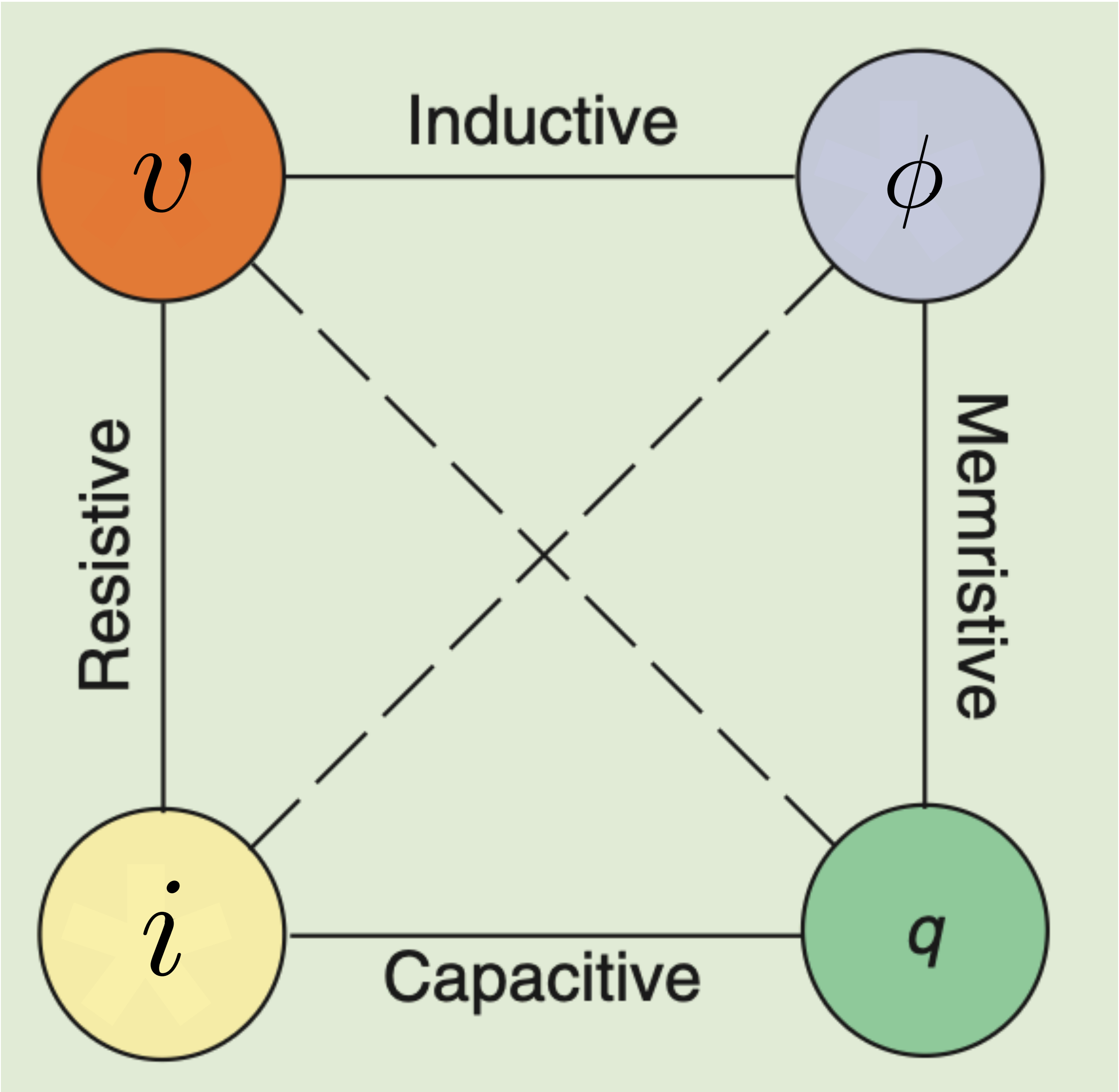
$$\int_{t_0}^{t} v(\tau)\, i(\tau)\, d\tau = \int_{t_0}^{t} R(x(\tau), i(\tau))\, i^2(\tau)\, d\tau \geqslant 0,$$

No energy discharge property

$$v(t)i(t) \geq 0 \qquad \text{for all } t$$

L.O. Chua, S.M.Kang, Memristive devices and systems
Proceeding of the IEEE, Vol. 64, No.2 1976

Motivation 3: **state functions of a memristor? (Chua 1971, Jeltsema-Scherpen 2009)**



$$\phi = m(q)$$

$$v = \frac{d}{dt}\phi$$

$$i = \frac{d}{dt}q$$

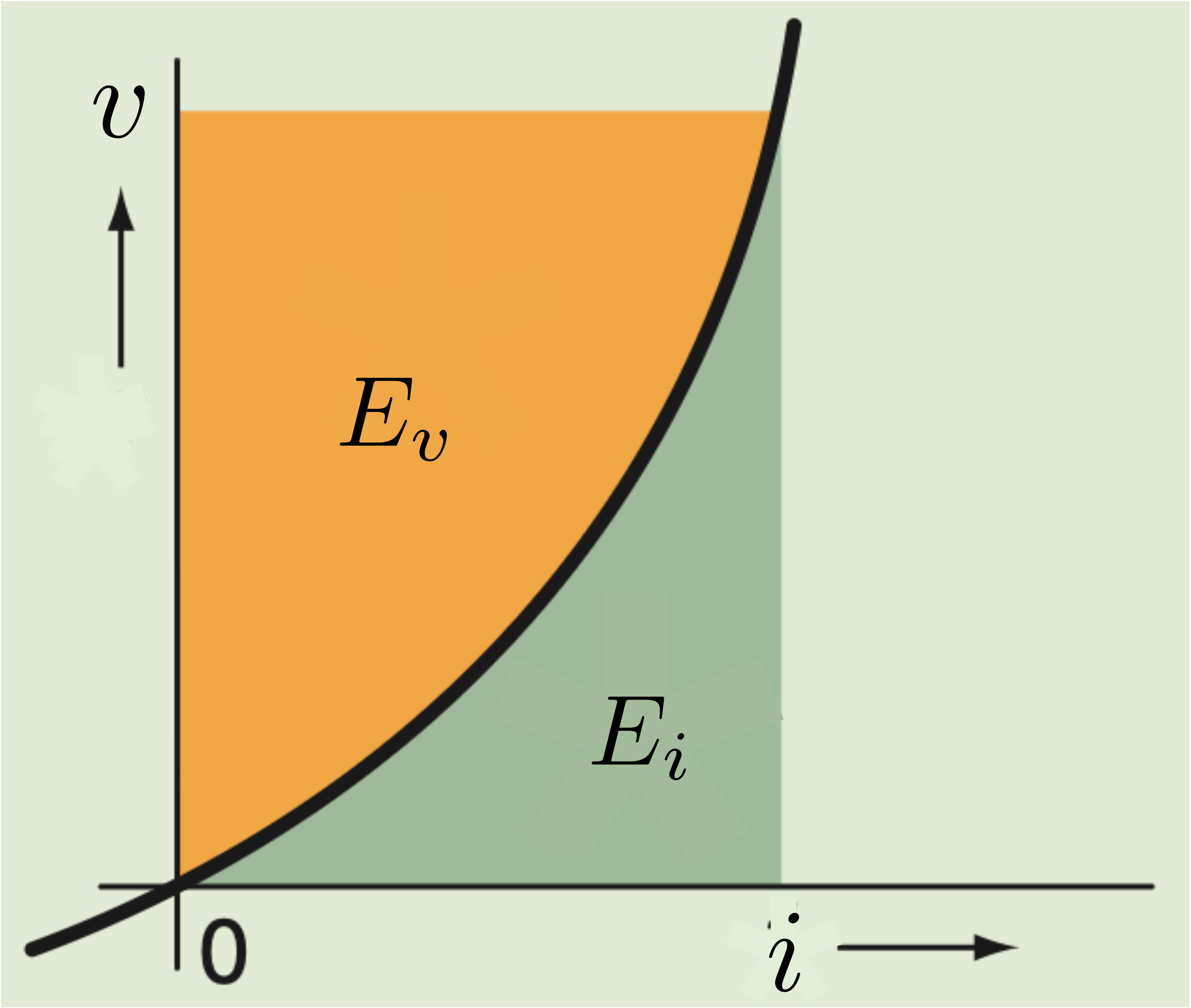
$$v = M(q)i = M\left(\int i\right)i$$

$$M(q) := \frac{\partial m(q)}{\partial q}$$

$$v = M(x)i \quad \dot{x} = i$$

D. Jeltsema, J. Scherpen, Multidomain modeling of nonlinear network and systems,
IEEE Control Systems Magazine, 2009

Motivation 3: **state functions of a memristor? (Chua 1971, Jeltsema-Scherpen 2009)**



Content and co-content

$$i = g(v) = \frac{\partial}{\partial_v} E_v$$

$$E_v = \int_0^v i dv = \int_0^v g(v) dv$$

$$v = r(i) = \frac{\partial}{\partial i} E_i$$

$$E_i = \int_0^i v\, di = \int_0^i r(i) di$$

$$E_i = E_v = \frac{1}{2} R i^2$$

D. Jeltsema, J. Scherpen, Multidomain modeling of nonlinear network and systems, IEEE Control Systems Magazine, 2009

Part 1: Motivation

Instantaneous i-v relation
that depends on the past

$$\dot{x} = f(x, u, t)$$
$$y = g(x, u, t)u$$

Physics of ODEs with 'internal' variables? Why?

Physics fundamentals: passivity, power dissipation, content and co-content

$$\int_{t_0}^{t} v(\tau)\, i(\tau)\, d\tau \geqslant 0\,, \qquad v(t)i(t) \geq 0$$

state functions of a memristor?

Hodgkin-Huxley neurons (mem-circuit)

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Memristive element

Operator-theoretic view

$$\mathbf{i}(t) \;=\; \mathbf{g}(\mathbf{v})(t)\,\mathbf{v}(t), \qquad \mathbf{v} \in \mathcal{L}_{2(-\infty,T]}$$

$$i \;=\; g(\mathbf{v})\,v.$$

Instantaneous but past-dependent i-v relation

Assumption 1: [Fading memory]

Input signals that differ in the remote past have similar current outputs

Assumption 2: [Energy dissipation] $\mathbf{g}(\mathbf{v})(t) \geq \varepsilon > 0,$

$$\langle \mathbf{i}, \mathbf{v} \rangle = \int_{-\infty}^{T} \mathbf{i}(t)\mathbf{v}(t)dt = \int_{-\infty}^{T} \mathbf{v}(t)\mathbf{g}(\mathbf{v})(t)\mathbf{v}(t)dt = \langle \mathbf{v}, \mathbf{v} \rangle_{\mathbf{g}} \geq 0$$

Dissipated energy is the Riemannian metric

From a geometric perspective, if we endow $\mathcal{L}_{2(-\infty,T]}$ with the structure of a Riemannian manifold $(\mathcal{L}_{2(-\infty,T]}, \mathbf{g})$, the dissipated energy of a memristive element corresponds to the quadratic norm $\|\mathbf{v}\|_g^2$ associated to the weighted inner product

$$\langle \mathbf{v}_1, \mathbf{v}_2 \rangle_g = \int_{-\infty}^{T} \mathbf{v}_1(t)\mathbf{g}(\mathbf{v})(t)\mathbf{v}_2(t)\mathrm{d}t \geq 0,$$

where $\mathbf{v}_1$ and $\mathbf{v}_2 \in T_{\mathbf{v}}\mathcal{L}_{2(-\infty,T]}$ are generic tangent vectors. This interpretation of the memconductance as a Riemannian metric is the basis of the gradient modelling in the next section.

Linear resistor

$$i = gv$$

Dissipated energy

$$\mathcal{D}(v) := \langle i, v \rangle = gv^2 = \langle v, v \rangle_g$$

Co-content

$$\mathcal{E}(\bar{v}) := \int_0^{\bar{v}} \langle i, \mathrm{d}v \rangle = \int_0^{\bar{v}} gv\mathrm{d}v = \frac{1}{2} \langle \bar{v}, \bar{v} \rangle_g$$

The co-content <u>determines</u> the resistive relationship

$$\langle \operatorname{grad} \mathcal{E}(v), v \rangle = D_v \mathcal{E}(v) = gv^2$$

$$i = \operatorname{grad} \mathcal{E}(v) = gv.$$

Nonlinear resistor

$$i = g(v)v$$

Dissipated energy

$$\mathcal{D}(v) := \langle i, v \rangle = g(v)v^2 = \langle v, v \rangle_g$$

Co-content

$$\mathcal{E}(\bar{v}) := \int_0^{\bar{v}} \langle i, \mathrm{d}v \rangle = \int_0^{\bar{v}} g(v)v\mathrm{d}v \neq \frac{1}{2} \langle \bar{v}, \bar{v} \rangle_g$$

The co-content <u>determines</u> the resistive relationship

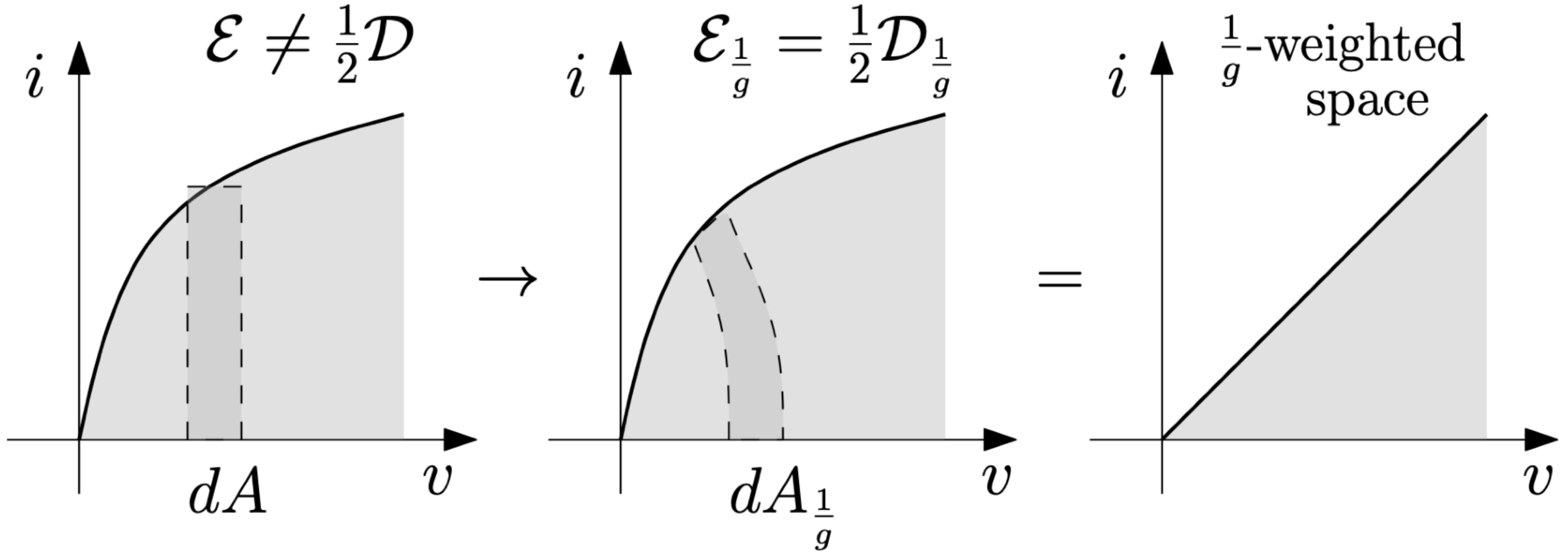
$$\langle \operatorname{grad} \mathcal{E}(v), v \rangle = D_v \mathcal{E}(v) = g(v)v^2$$

$$i = \operatorname{grad} \mathcal{E}(v)$$

Nonlinear resistor

The linear equivalent between dissipation and co-content is recovered by taking the voltage space as a Riemannian manifold with metric 1/g

$$i = g(v)v$$



$$\mathcal{E}(\bar{v}) := \int_0^{\bar{v}} \langle i, \mathrm{d}v \rangle = \int_0^{\bar{v}} g(v)v\mathrm{d}v \neq \frac{1}{2}\langle \bar{v}, \bar{v} \rangle_g$$

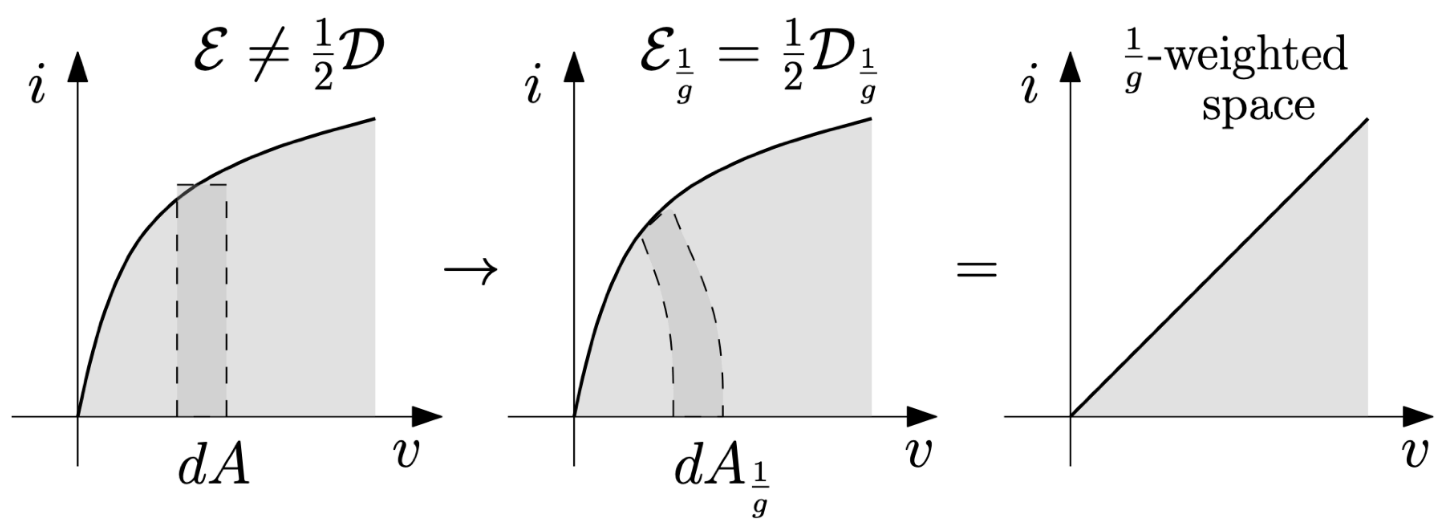
1/g - dissipated energy

$$\mathcal{D}_{\frac{1}{g}}(v) := \langle i, v \rangle_{\frac{1}{g}} = v^2 = \langle v, v \rangle$$

1/g co-content

$$\mathcal{E}_{\frac{1}{g}}(\bar{v}) := \int_0^{\bar{v}} \langle i, \mathrm{d}v \rangle_{\frac{1}{g}} = \int_0^{\bar{v}} v\mathrm{d}v = \frac{1}{2}\bar{v}^2 = \frac{1}{2}\langle \bar{v}, \bar{v} \rangle$$

Nonlinear resistor (on the Rimannian manifold with metric 1/g)



$$i = g(v)v$$

1/g - dissipated energy

$$\mathcal{D}_{\frac{1}{g}}(v) := \langle i, v \rangle_{\frac{1}{g}} = v^2 = \langle v, v \rangle$$

1/g co-content

$$\mathcal{E}_{\frac{1}{g}}(\bar{v}) := \int_0^{\bar{v}} \langle i, \mathrm{d}v \rangle_{\frac{1}{g}} = \int_0^{\bar{v}} v \mathrm{d}v = \frac{1}{2}\bar{v}^2 = \frac{1}{2}\langle \bar{v}, \bar{v} \rangle$$

The 1/g co-content <u>determines</u> the resistive relationship

$$\left\langle \mathrm{grad}_{\frac{1}{g}} \mathcal{E}_{\frac{1}{g}}(v), v \right\rangle_{\frac{1}{g}} = D_v \mathcal{E}_{\frac{1}{g}}(v) = v^2$$

$$i = \mathrm{grad}_{\frac{1}{g}} \mathcal{E}_{\frac{1}{g}}(v) = g(v)v$$

Memristor

$$\mathbf{i}(t) = \mathbf{g}(\mathbf{v})(t)\,\mathbf{v}(t),$$

Inner products

$$\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = \int_{-\infty}^{T} \mathbf{v}_1(t)\mathbf{v}_2(t)\mathrm{d}t \qquad \langle \mathbf{v}_1, \mathbf{v}_2 \rangle_{\frac{1}{\mathbf{g}}} = \int_{-\infty}^{T} \frac{\mathbf{v}_1(t)\mathbf{v}_2(t)}{\mathbf{g}(\mathbf{v})(t)}\mathrm{d}t$$

1/g - dissipated energy

$$\mathcal{D}_{\frac{1}{\mathbf{g}}} = \langle \mathbf{v}, \mathbf{v} \rangle$$

1/g co-content

$$\mathcal{E}_{\frac{1}{\mathbf{g}}} = \frac{1}{2}\langle \mathbf{v}, \mathbf{v} \rangle$$

The 1/g co-content <u>determines</u> the resistive relationship

$$\left\langle \mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}), \mathbf{v} \right\rangle_{\frac{1}{\mathbf{g}}} = \mathbf{D}_{\mathbf{v}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) = \langle \mathbf{v}, \mathbf{v} \rangle, \qquad \mathbf{i} = \mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) = \mathbf{g}(\mathbf{v})(\cdot)\mathbf{v}(\cdot)$$

Linear resistor

$$i(t) = gv(t)$$

$$\mathcal{E}_{\frac{1}{g}}(\bar{v}) := \frac{1}{2}\langle \bar{v}, \bar{v} \rangle$$

$$i = \mathrm{grad}_{\frac{1}{g}}\, \mathcal{E}_{\frac{1}{g}}(v)$$

Memristor

$$\mathbf{i}(t) \;=\; \mathbf{g}(\mathbf{v})(t)\,\mathbf{v}(t),$$

$$\mathcal{E}_{\frac{1}{\mathbf{g}}} \;=\; \tfrac{1}{2}\langle \mathbf{v}, \mathbf{v} \rangle$$

$$\mathbf{i} = \mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v})$$

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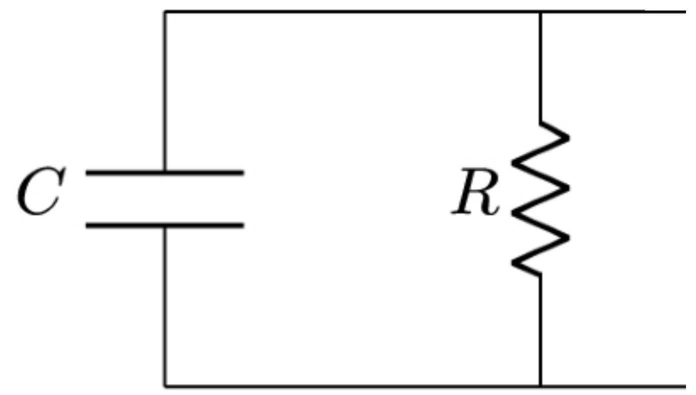
Linear RC



Dynamics

$$C\dot{v} = -gv + i = -\operatorname{grad}\mathcal{E}(v) + \operatorname{grad}\langle v, i\rangle$$

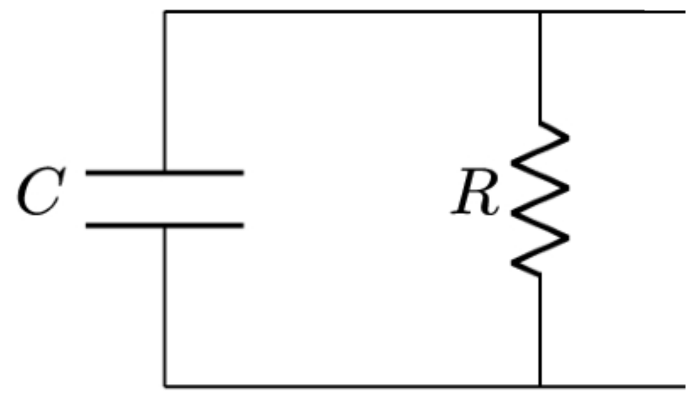
Potential (constant currents)

$$\mathcal{P}(v) = \mathcal{E}(v) - \langle v, i\rangle$$

Convergence

$$\dot{\mathcal{P}}(v) = D_{\dot{v}}\mathcal{P}(v) = \langle \operatorname{grad}\mathcal{P}(v), \dot{v}\rangle = -\gamma \operatorname{grad}\mathcal{P}(v)^2$$

Linear RC



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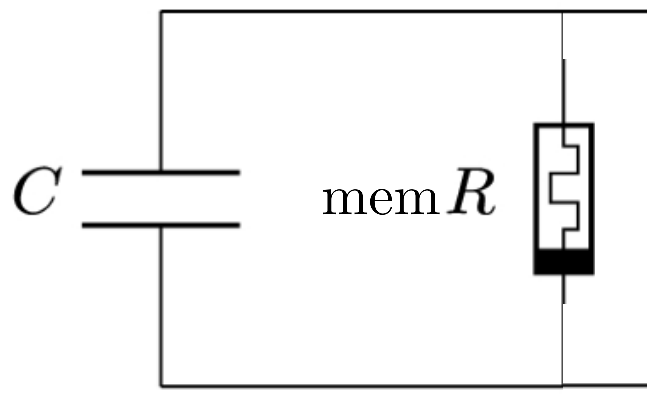
mem-RC



$$\mathcal{E}_{\frac{1}{\mathbf{g}}} = \frac{1}{2}\langle \mathbf{v}, \mathbf{v}\rangle$$

Dynamics

$$C\dot{v} = -\operatorname{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) + \operatorname{grad}\langle \mathbf{i}, \mathbf{v}\rangle$$

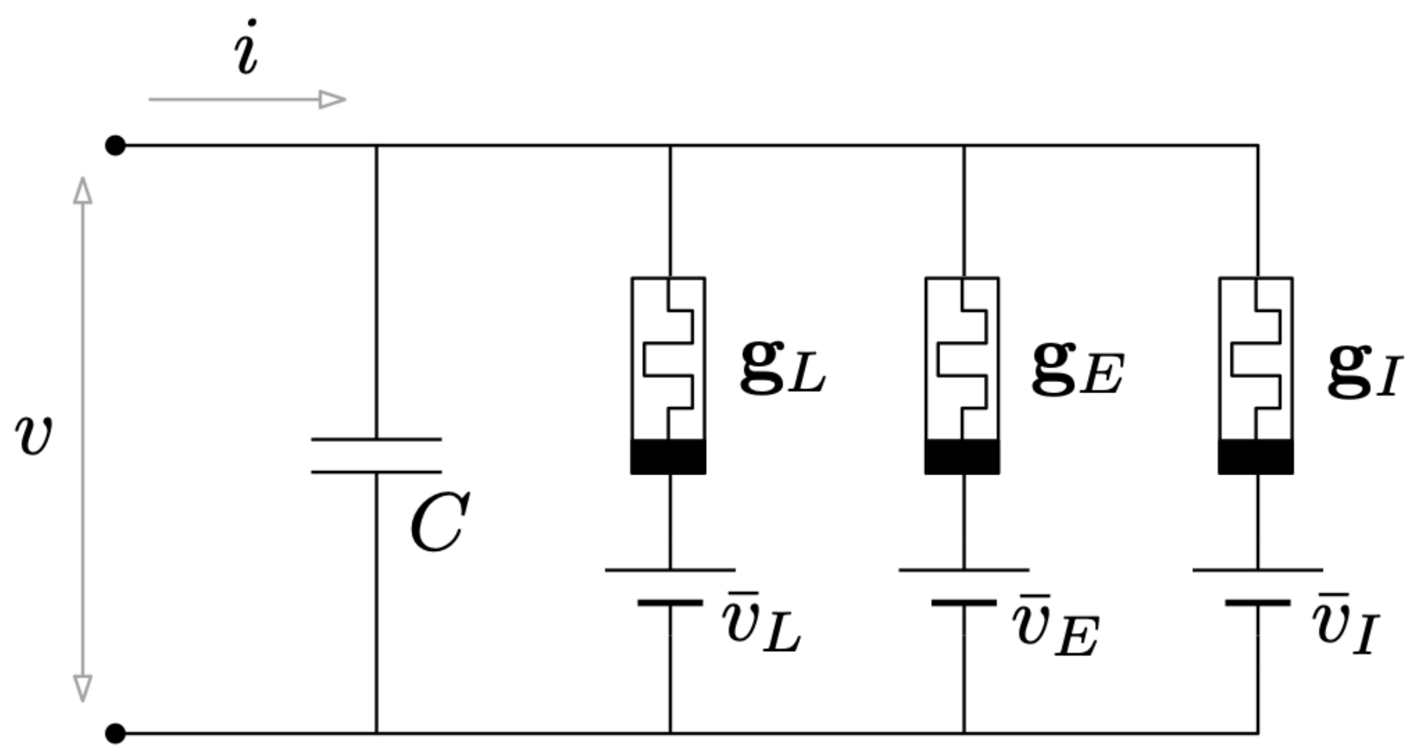
Energy dissipation

$$\mathcal{S}_C(\mathbf{v}(t)) = \tfrac{1}{2}C\mathbf{v}(t)^2.$$

$$\begin{aligned} S_C(\mathbf{v}(T)) - S_C(\mathbf{v}(-\infty)) &= \langle \mathbf{v}, C\dot{\mathbf{v}}\rangle \\ &= \left\langle \mathbf{v}, -\mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) + \mathbf{i}\right\rangle \\ &\leq \langle \mathbf{i}, \mathbf{v}\rangle \end{aligned}$$

for closed trajectories $\left\langle \mathbf{v}, -\mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v}) + \mathbf{i}\right\rangle = 0.$

Hodgkin-Huxley gradient memcircuit



$$C\dot{v} = -i_L - i_E - i_I + i$$

$$i_L = g_L(\mathbf{v} - \bar{\mathbf{v}}_L)(v - \bar{v}_L)$$
$$i_E = g_E(\mathbf{v} - \bar{\mathbf{v}}_E)(v - \bar{v}_E)$$
$$i_I = g_I(\mathbf{v} - \bar{\mathbf{v}}_I)(v - \bar{v}_I)$$

Gradient view

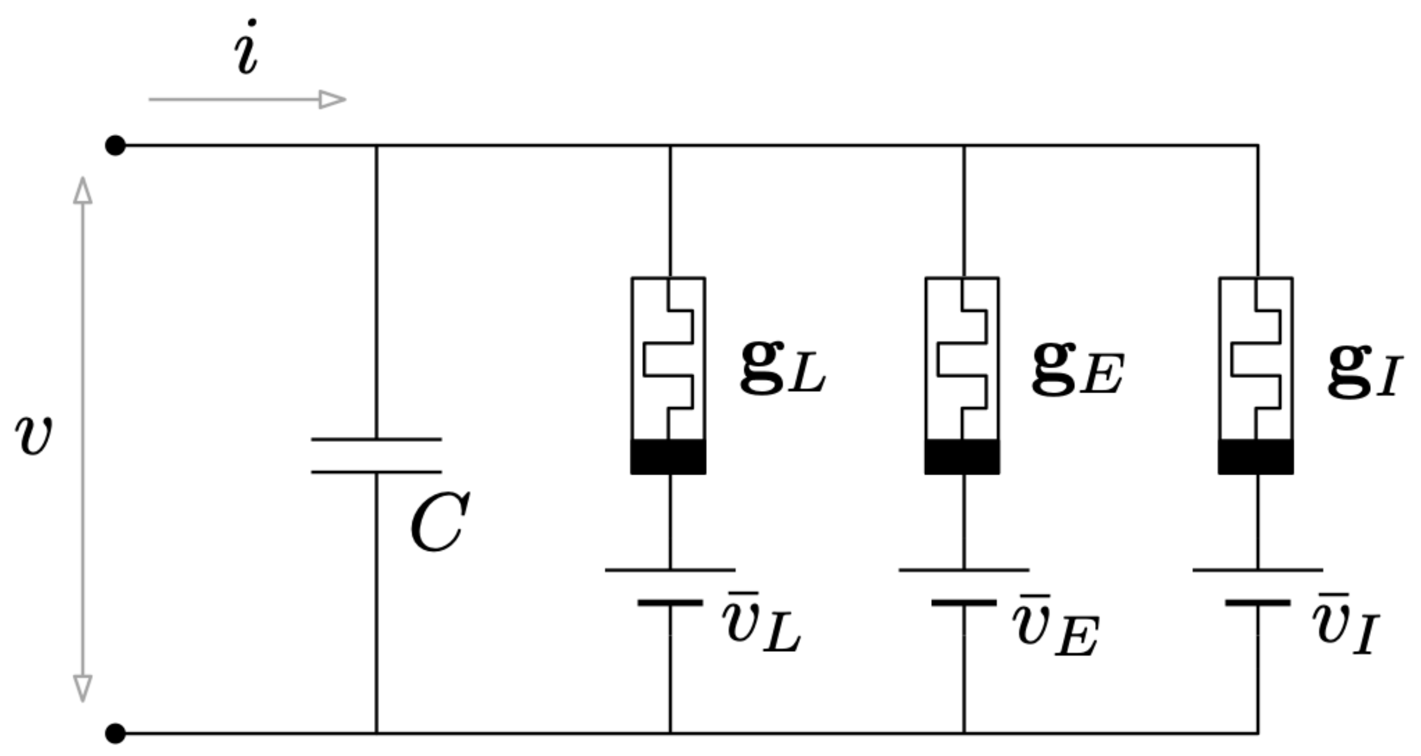
$$\mathcal{E}_{\frac{1}{\mathbf{g}_L}} = \mathcal{E}_{\frac{1}{\mathbf{g}_E}} = \mathcal{E}_{\frac{1}{\mathbf{g}_I}} = \frac{1}{2}\langle \mathbf{v}, \mathbf{v} \rangle \quad \Rightarrow \quad \begin{aligned} C\dot{v} = &-\mathrm{grad}_{\frac{1}{\mathbf{g}_L}} \mathcal{E}_{\frac{1}{\mathbf{g}_L}}(\mathbf{v} - \bar{\mathbf{v}}_L) - \mathrm{grad}_{\frac{1}{\mathbf{g}_E}} \mathcal{E}_{\frac{1}{\mathbf{g}_E}}(\mathbf{v} - \bar{\mathbf{v}}_E) \\ &- \mathrm{grad}_{\frac{1}{\mathbf{g}_I}} \mathcal{E}_{\frac{1}{\mathbf{g}_I}}(\mathbf{v} - \bar{\mathbf{v}}_I) + \mathrm{grad}\,\langle \mathbf{v}, \mathbf{i} \rangle\,. \end{aligned}$$

Network view

Take now $\mathbb{1}^\mathsf{T} = [1, 1, 1]$ and define

$$\begin{aligned} \bar{\mathbf{V}}^\mathsf{T} &= \begin{bmatrix} \bar{\mathbf{v}}_L & \bar{\mathbf{v}}_E & \bar{\mathbf{v}}_I \end{bmatrix}, \\ \frac{1}{\mathbf{G}} &= \mathrm{diag}\left(\frac{1}{\mathbf{g}_L}, \frac{1}{\mathbf{g}_E}, \frac{1}{\mathbf{g}_I}\right), \\ \mathcal{E}_{\frac{1}{\mathbf{G}}} &= \mathrm{diag}\left(\mathcal{E}_{\frac{1}{\mathbf{g}_L}}, \mathcal{E}_{\frac{1}{\mathbf{g}_E}}, \mathcal{E}_{\frac{1}{\mathbf{g}_I}}\right). \end{aligned} \quad \Rightarrow \quad C\dot{v} = \mathbb{1}^\mathsf{T}\,\mathrm{grad}_{\frac{1}{\mathbf{G}}} \mathcal{E}_{\frac{1}{\mathbf{G}}}(\mathbb{1}\mathbf{v} - \bar{\mathbf{V}}) + \mathrm{grad}\,\langle \mathbf{v}, \mathbf{i} \rangle$$

Hodgkin-Huxley gradient memcircuit



$$C\dot{v} = -i_L - i_E - i_I + i$$

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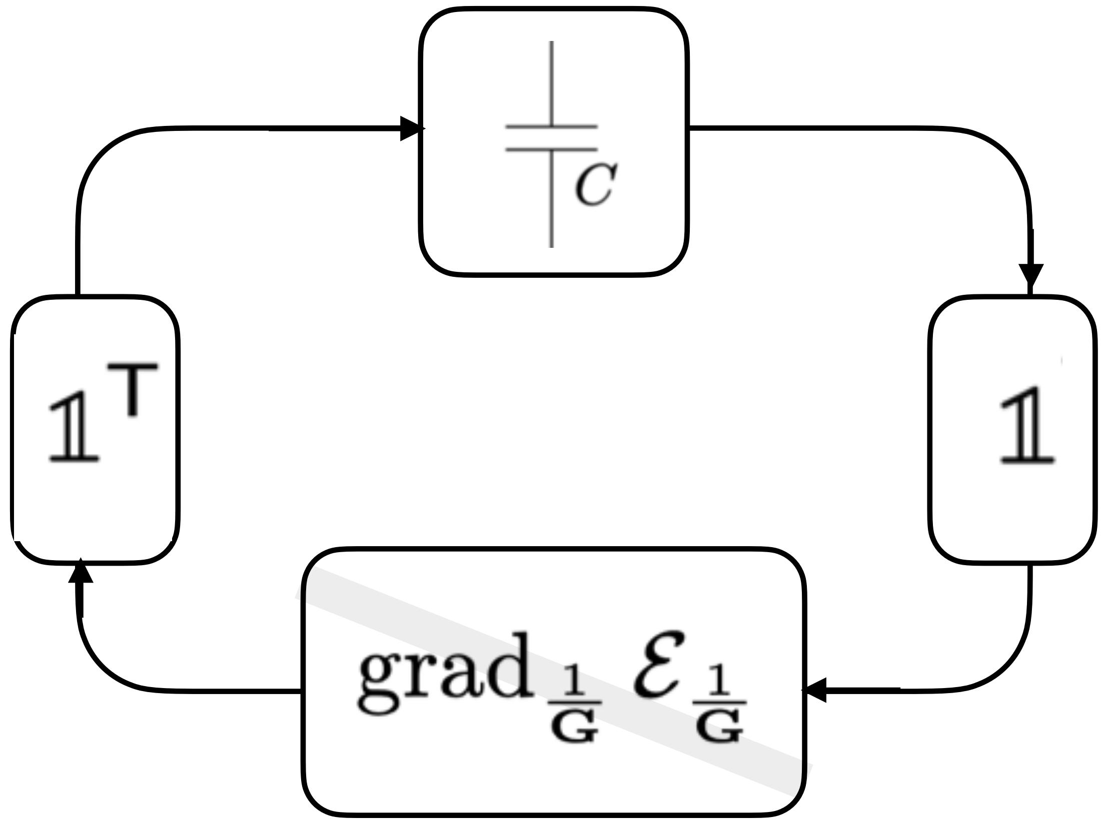
Gradient view

$$\mathcal{E}_{\frac{1}{\mathbf{g}_L}} = \mathcal{E}_{\frac{1}{\mathbf{g}_E}} = \mathcal{E}_{\frac{1}{\mathbf{g}_I}} = \frac{1}{2}\langle \mathbf{v}, \mathbf{v}\rangle$$



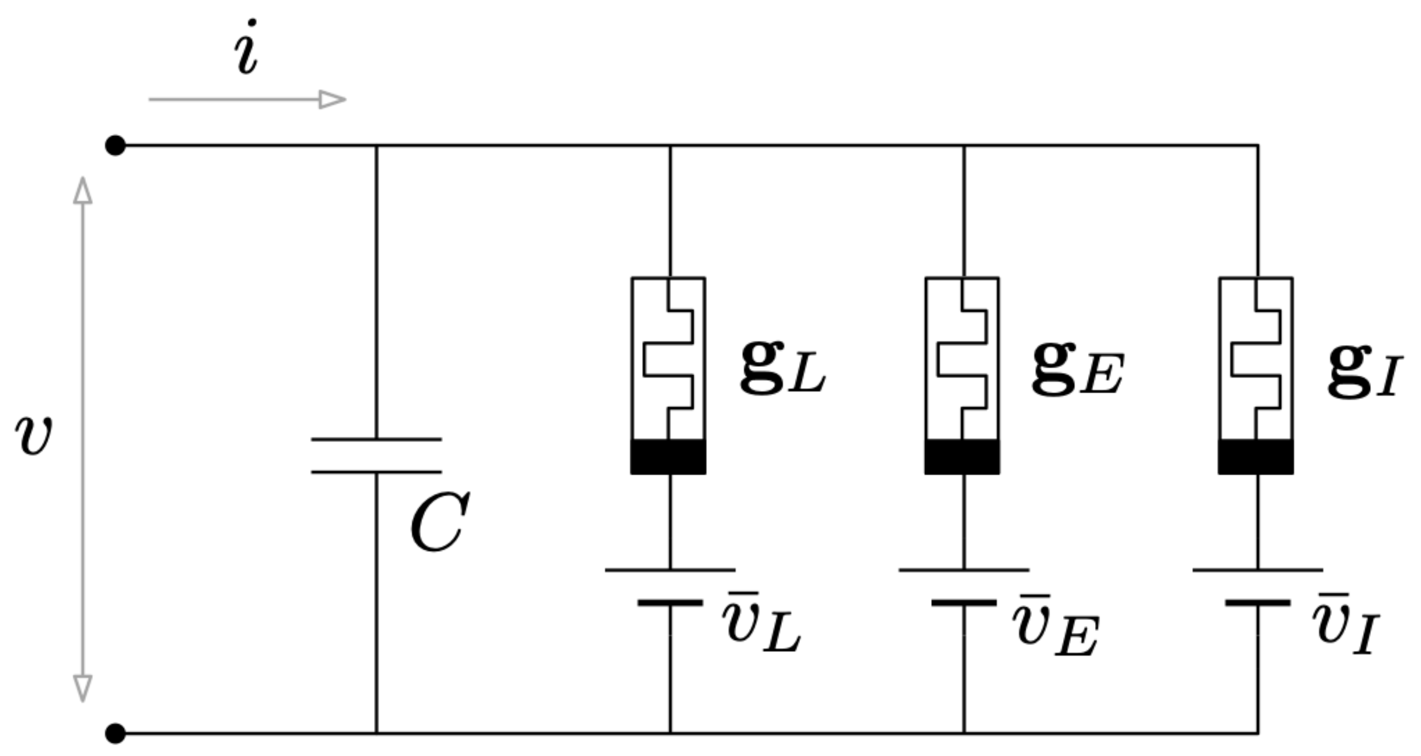
Take now $\mathbb{1}^\mathsf{T} = [1, 1, 1]$ and define

$$\bar{\mathbf{V}}^\mathsf{T} = \begin{bmatrix} \bar{\mathbf{v}}_L & \bar{\mathbf{v}}_E & \bar{\mathbf{v}}_I \end{bmatrix},$$
$$\frac{1}{\mathbf{G}} = \mathrm{diag}\left(\frac{1}{\mathbf{g}_L}, \frac{1}{\mathbf{g}_E}, \frac{1}{\mathbf{g}_I}\right),$$
$$\mathcal{E}_{\frac{1}{\mathbf{G}}} = \mathrm{diag}\left(\mathcal{E}_{\frac{1}{\mathbf{g}_L}}, \mathcal{E}_{\frac{1}{\mathbf{g}_E}}, \mathcal{E}_{\frac{1}{\mathbf{g}_I}}\right).$$

Network view

$$C\dot{v} = \mathbb{1}^\mathsf{T}\,\mathrm{grad}_{\frac{1}{\mathbf{G}}}\,\mathcal{E}_{\frac{1}{\mathbf{G}}}(\mathbb{1}\mathbf{v} - \bar{\mathbf{V}}) + \mathrm{grad}\,\langle \mathbf{v}, \mathbf{i}\rangle$$

Hodgkin-Huxley gradient memcircuit

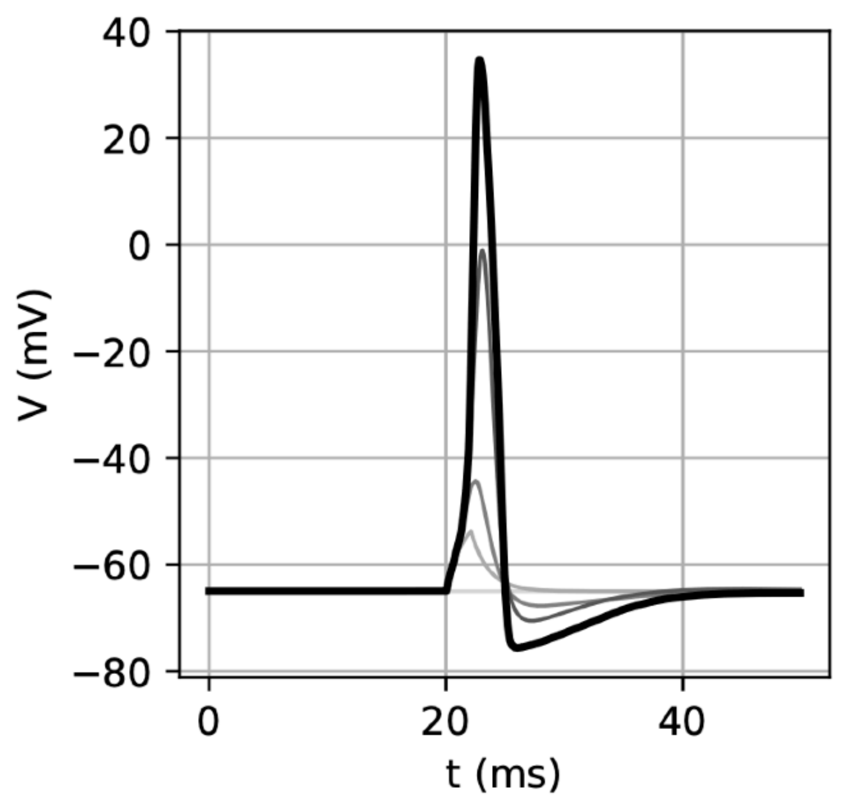


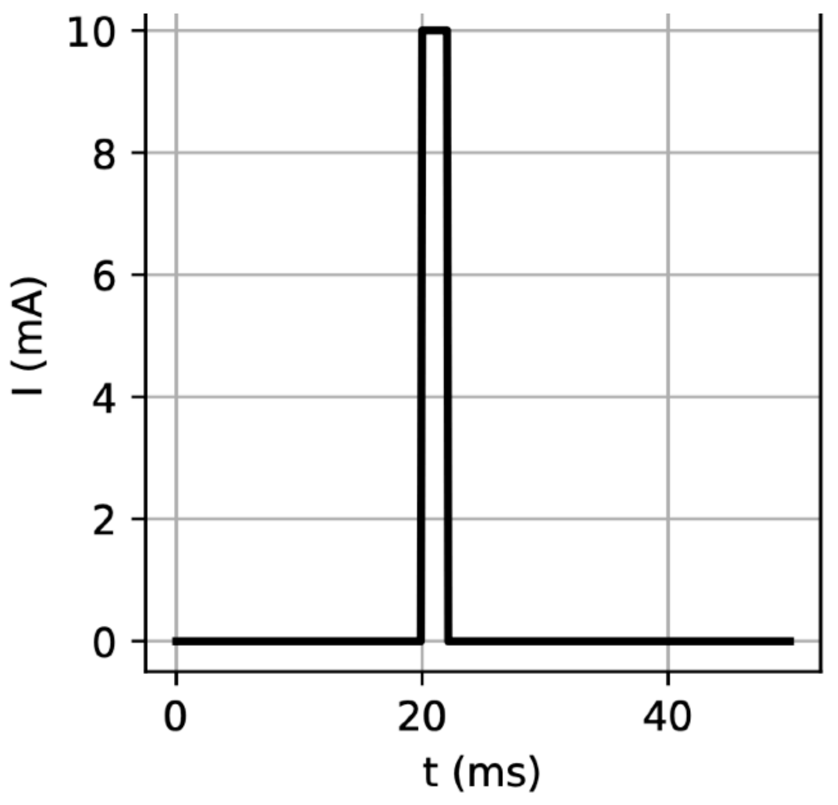
$$C\dot{v} = -i_L - i_E - i_I + i$$

$$i_L = g_L(\mathbf{v} - \bar{\mathbf{v}}_L)(v - \bar{v}_L)$$
$$i_E = g_E(\mathbf{v} - \bar{\mathbf{v}}_E)(v - \bar{v}_E)$$
$$i_I = g_I(\mathbf{v} - \bar{\mathbf{v}}_I)(v - \bar{v}_I)$$

What representation for conductances? (why ODEs?)

Compute: zeros instead of integration steps
two-step iteration - (i) given v update G, (ii) given G update V





$$C\dot{v} = \mathbb{1}^{\mathsf{T}}\,\mathrm{grad}_{\frac{1}{\mathbf{G}}}\,\mathcal{E}_{\frac{1}{\mathbf{G}}}(\mathbb{1}\mathbf{v} - \bar{\mathbf{V}}) + \mathrm{grad}\,\langle \mathbf{v}, \mathbf{i} \rangle$$

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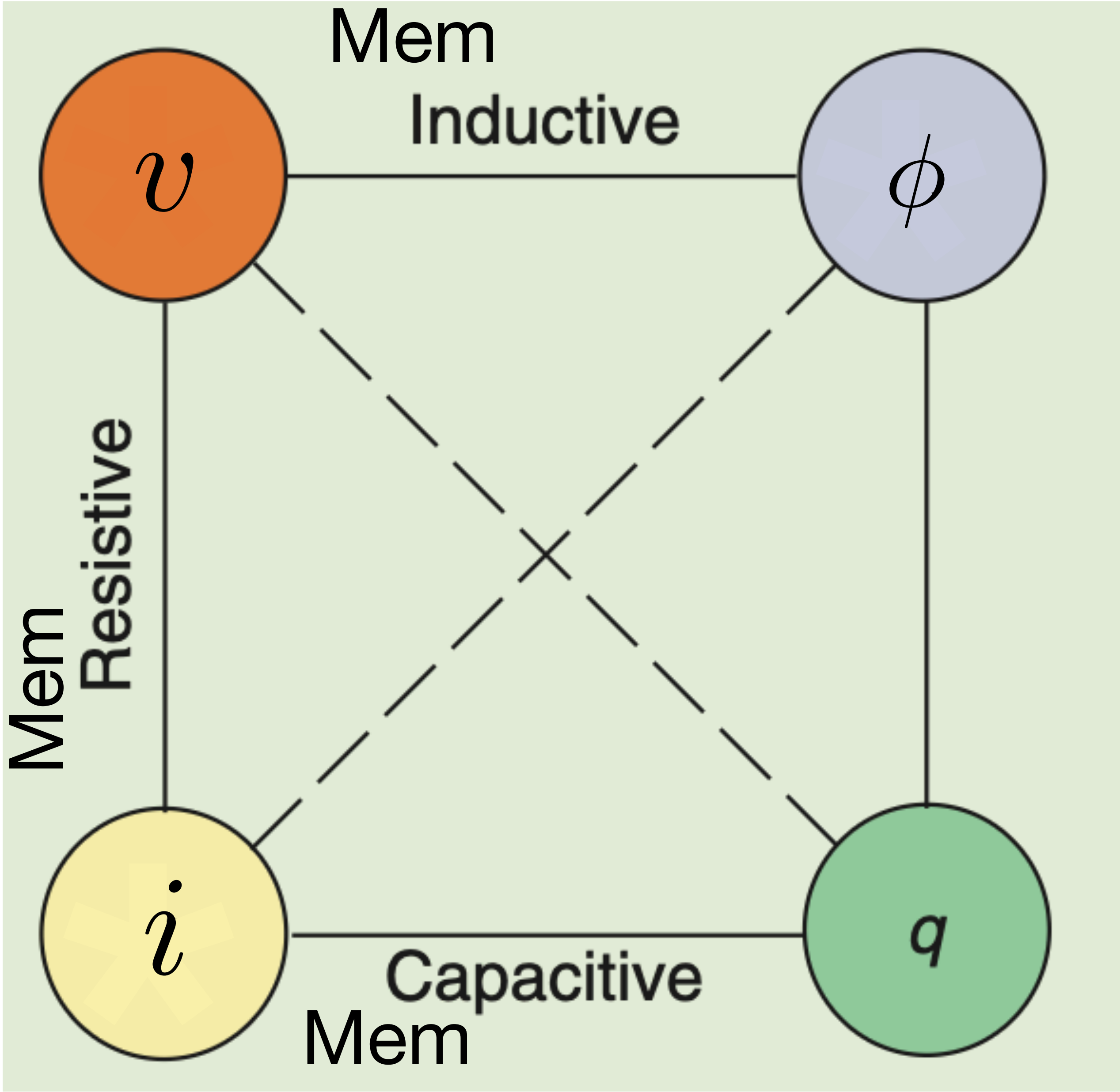
Mem-elements (a proposal)



D. Jeltsema, J. Scherpen, Multidomain modeling of nonlinear network and systems,
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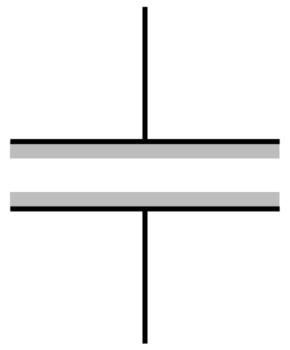
Memcapacitor



$$q = c(\mathbf{v})v \qquad \dot{q} = i \qquad i = \dot{c}(\mathbf{v})v + c(\mathbf{v})\dot{v}$$

$$c(\mathbf{v})\dot{v} = -\dot{c}(\mathbf{v})v + i.$$

1/g co-content

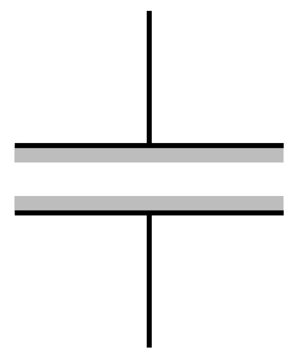
$$\mathcal{E}^*_{\frac{1}{\mathbf{c}}}(\bar{\mathbf{v}}) := \int_0^{\bar{\mathbf{v}}} \langle \mathbf{q}, \mathrm{d}\mathbf{v} \rangle_{\frac{1}{\mathbf{c}}} = \frac{1}{2}\langle \bar{\mathbf{v}}, \bar{\mathbf{v}} \rangle$$

The 1/g co-content determines the resistive relationship

$$\mathbf{q} = \mathbf{grad}_{\frac{1}{\mathbf{c}}} \mathcal{E}^*_{\frac{1}{\mathbf{c}}}(\mathbf{v})$$

Passivity?

Cyclo-passivity of the nonlinear capacitor



$$q = c(v)v \qquad \dot{q} = i \qquad i = c(v)\dot{v} + \dot{c}(v)v$$

$$S(v) := \int_0^v \langle c(u)u, \mathrm{d}u \rangle \qquad \dot{S} = \partial_v \int_0^v \langle c(\bar{v})\bar{v}, \mathrm{d}\bar{v} \rangle \, \dot{v} = c(v)v\dot{v} = vi - \dot{c}(v)v^2$$

$$0 = S(v(T)) - S(v(-\infty)) \;\; = \int_{-\infty}^{T} vi \, \mathrm{d}t + H(v(T)) - H(v(-\infty)) \;\; = \int_{-\infty}^{T} vi \, \mathrm{d}t$$

$$H(v) = \int_0^v (\partial_u c(u)) \, u^2 \mathrm{d}u.$$

Lagrangian modeling of the nonlinear capacitor

$$L = \int_0^v \langle c(\bar{v})\bar{v}, \mathrm{d}\bar{v} \rangle \qquad i = \frac{\mathrm{d}}{\mathrm{d}t}\partial_v L = \frac{\mathrm{d}}{\mathrm{d}t}\left(c(v)v\right) = c(v)\dot{v} + \dot{c}(v)v = i$$

Cyclo-passive

$$L = \frac{1}{2}c(\phi)v^2$$

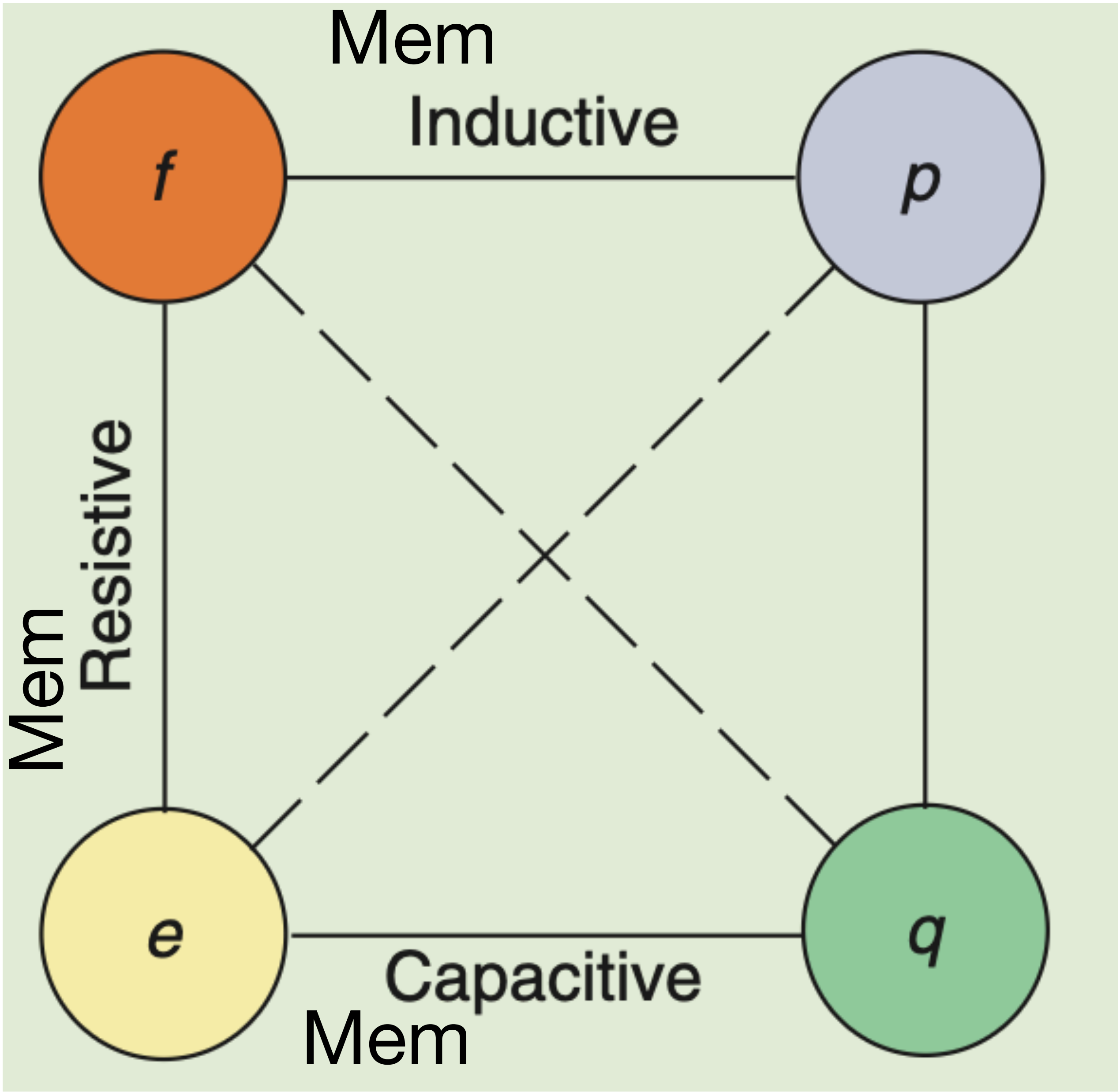
$$\phi = \int v$$

$$\begin{aligned} i &= \frac{\mathrm{d}}{\mathrm{d}t}\partial_v L - \partial_\phi L \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(c(\phi)v\right) - \frac{1}{2}\partial_\phi c(\phi)v^2 \\ &= \dot{c}(\phi)v + c(\phi)\dot{v} - \frac{1}{2}\partial_\phi c(\phi)v^2 \\ &= c(\phi)\dot{v} + \frac{1}{2}\partial_\phi c(\phi)v^2 \\ &= c(\phi)\dot{v} + \frac{1}{2}\dot{c}(\phi)v. \end{aligned}$$

Passive

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}c(\phi)v^2 &= \frac{1}{2}\dot{c}(\phi)v^2 + c(\phi)v\dot{v} \\ &= \frac{1}{2}\dot{c}(\phi)v^2 - \frac{1}{2}\dot{c}(\phi)v^2 + iv \\ &= iv. \end{aligned}$$

Mechanical mem-circuit (a proposal)



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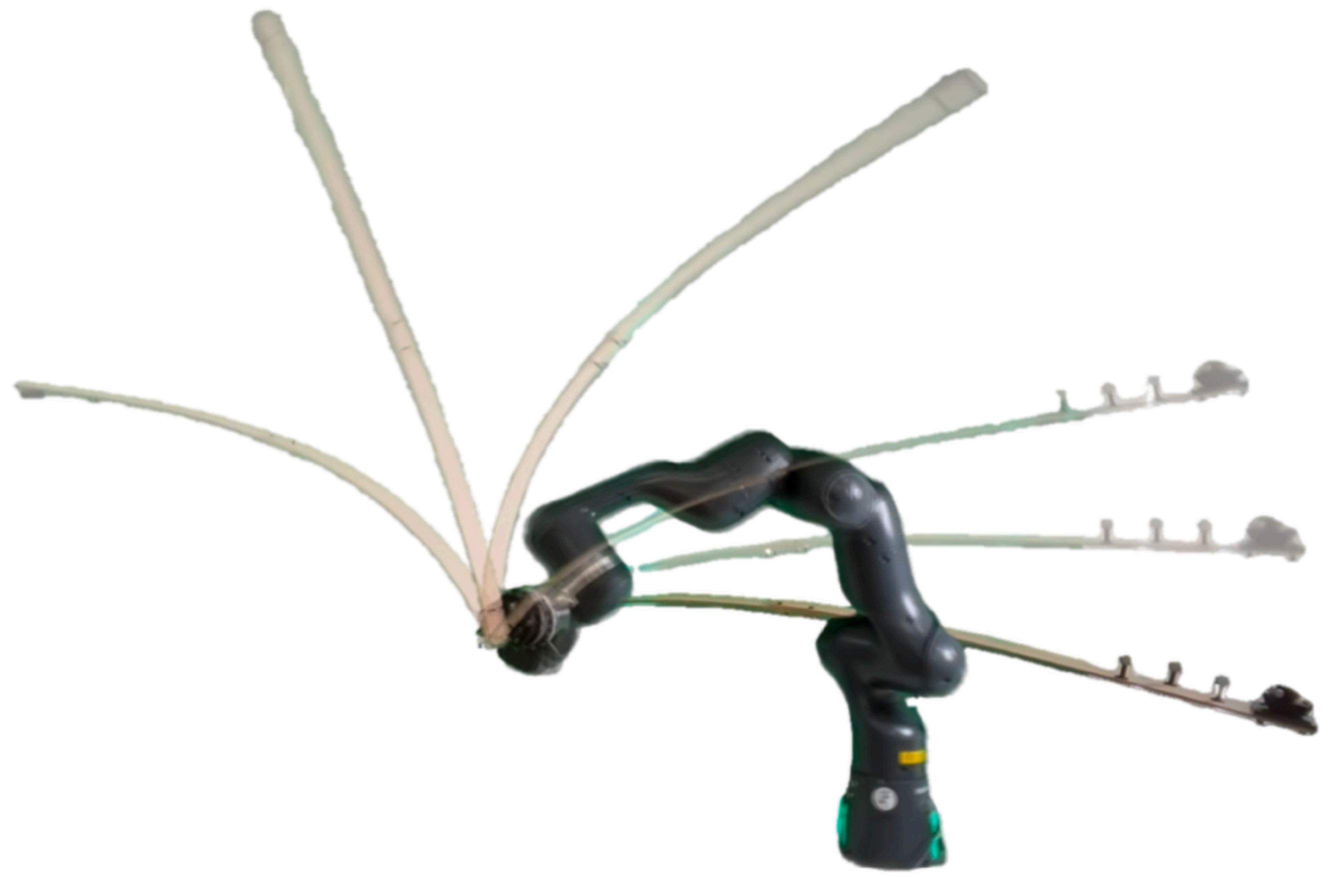
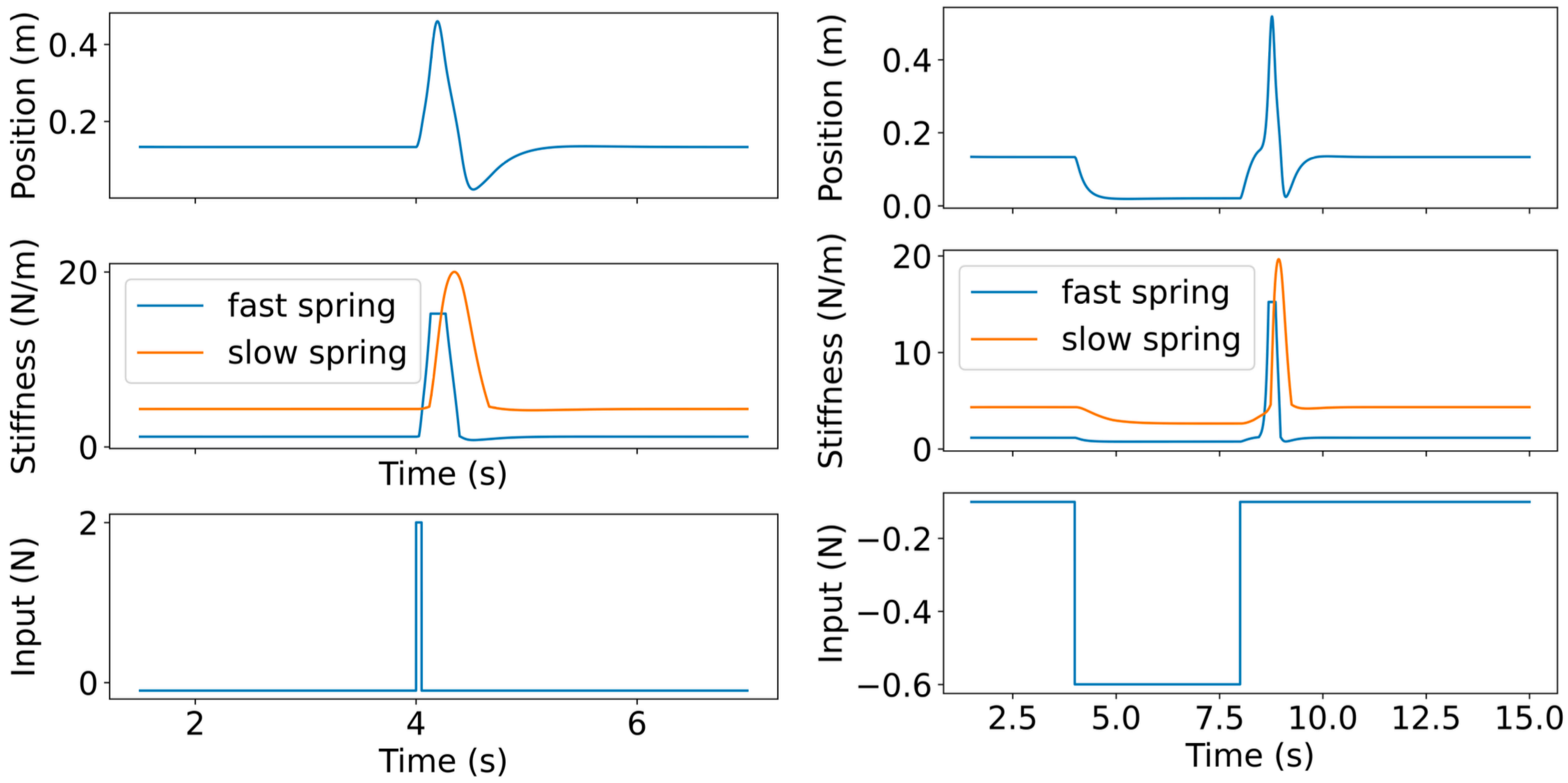
IEEE Control Systems Magazine, 2009

A mechanical spike

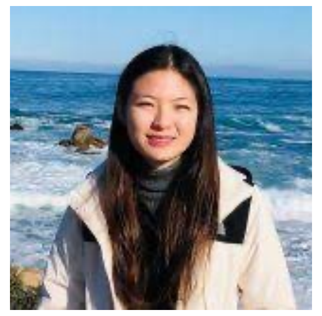
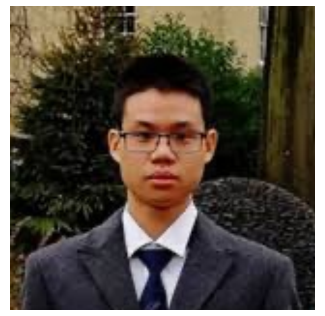
Y. Zhang and T. Huo

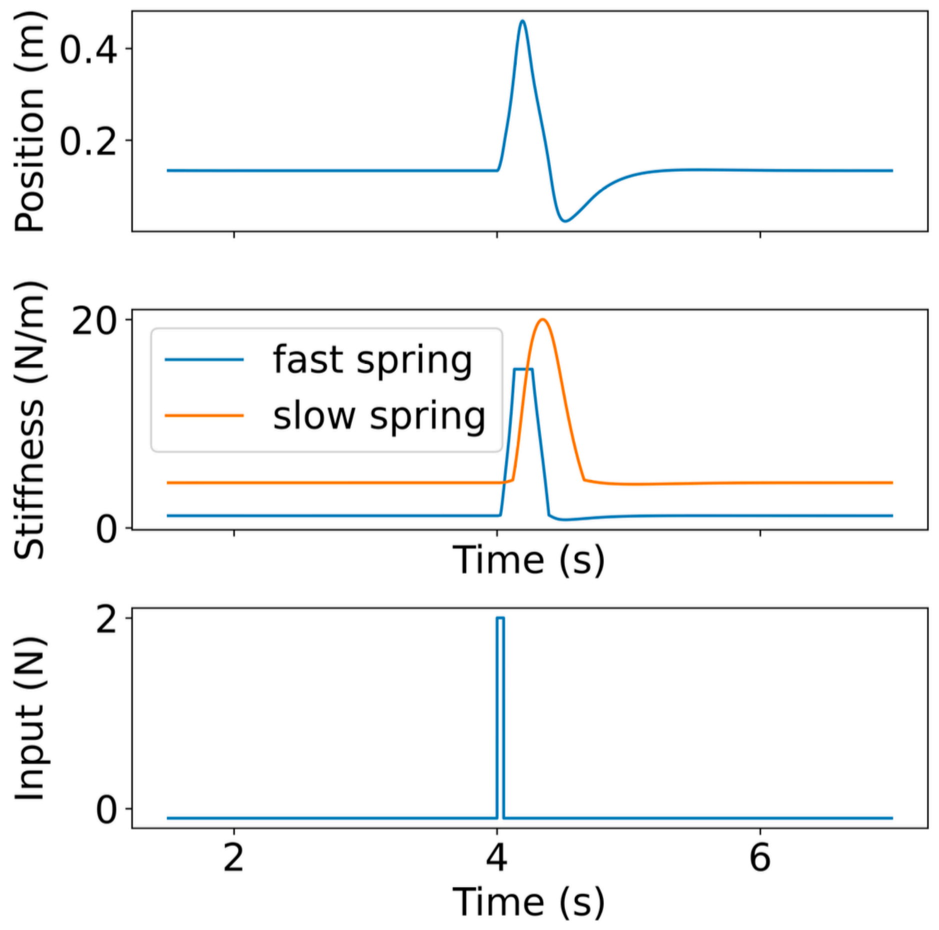


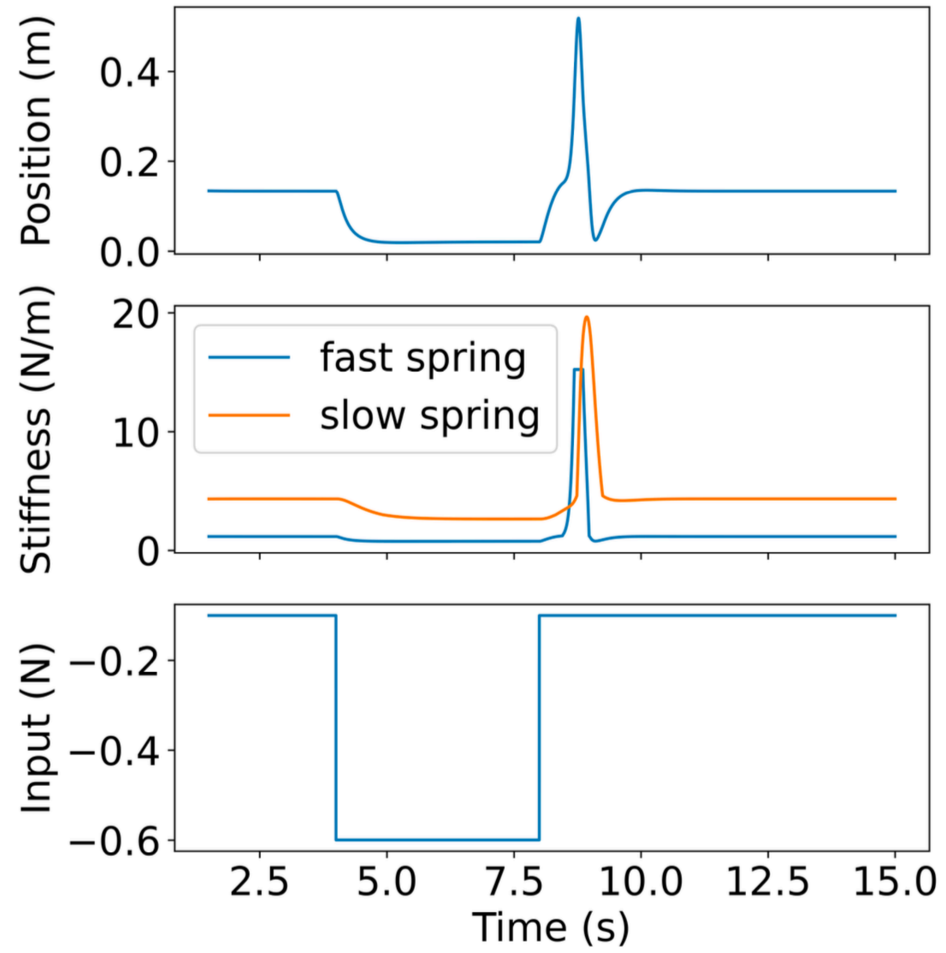


A mechanical spike

Y. Zhang and T. Huo





Thank you!

…and thanks to R. Sepulchre, Y. Zhang and T. Huo!

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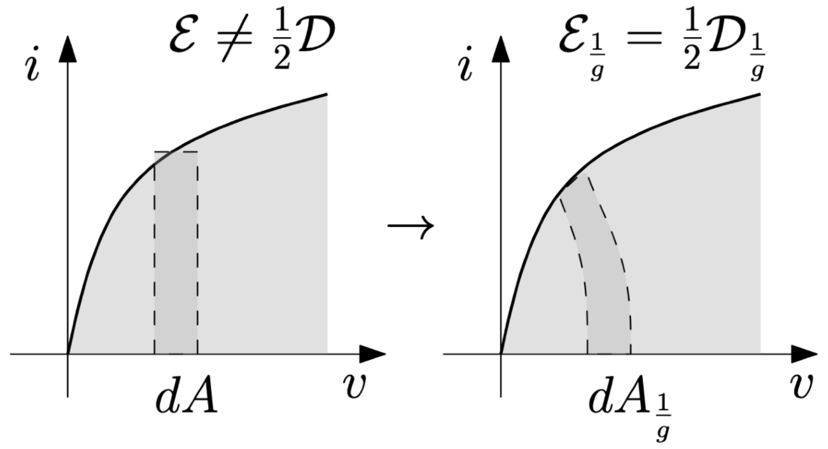
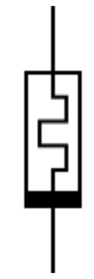


$$\mathbf{i}(t) \;=\; \mathbf{g}(\mathbf{v})(t)\,\mathbf{v}(t) \;=\mathbf{grad}_{\frac{1}{\mathbf{g}}}\mathcal{E}_{\frac{1}{\mathbf{g}}}(\mathbf{v})$$



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