University of Stuttgart
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SimTech

Physical Computation Workshop, Leuven, September 2025

Quantum computing through the lens of control

A tutorial introduction

Julian Berberich

Institute for Systems Theory and Automatic Control
University of Stuttgart

Potential of quantum computing

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*
PETER W. SHOR†

Universal Quantum Simulators
Seth Lloyd

Prime factorization:

- There exists no (known) efficient algorithm for classical computers
- Applications: cryptography

Quantum simulation:

- Simulating quantum systems is hard for classical computers
- Applications: drug design, quantum chemistry, material science

Quantum computing promises **major advances** in computations

Challenges in quantum computing

Noisy intermediate-scale quantum (NISQ) era

- Noisy devices → errors
- Scalability issues

Quantum Computing in the NISQ era and beyond

John Preskill

Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA

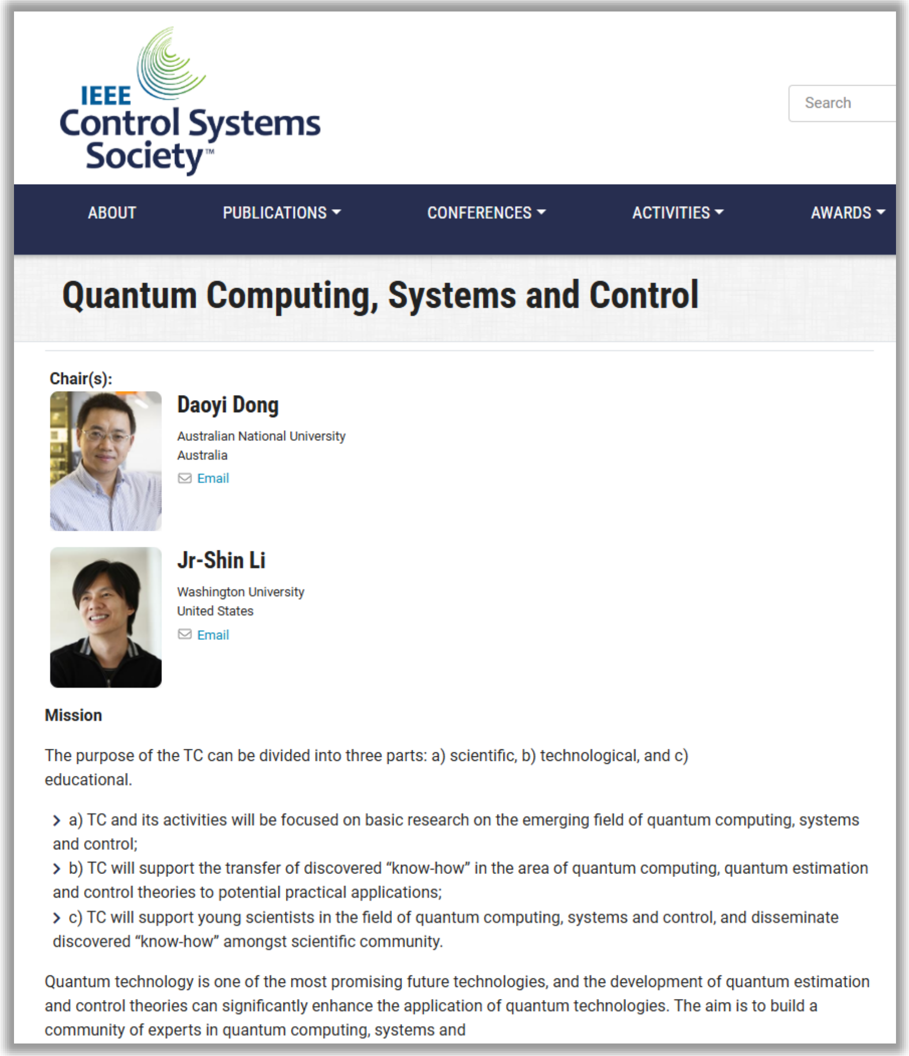
30 July 2018

- **Quantum computers exist** and can be **programmed & accessed via cloud**
- **Limited reliability & size due to errors**

Quantum computing faces a multitude of **theoretical and practical challenges**: Robustness, scalability, feedback, performance, convergence, optimality, …

Control-theoretic principles & methods **play a major role!**

Quantum computing and control



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Quantum Computing, Systems and Control

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United States
Email

Mission

The purpose of the TC can be divided into three parts: a) scientific, b) technological, and c) educational.

- a) TC and its activities will be focused on basic research on the emerging field of quantum computing, systems and control;
- b) TC will support the transfer of discovered "know-how" in the area of quantum computing, quantum estimation and control theories to potential practical applications;
- c) TC will support young scientists in the field of quantum computing, systems and control, and disseminate discovered "know-how" amongst scientific community.

Quantum technology is one of the most promising future technologies, and the development of quantum estimation and control theories can significantly enhance the application of quantum technologies. The aim is to build a community of experts in quantum computing, systems and

2025 IEEE International Conference on Quantum Control, Computing, and Learning (IEEE qCCL2025)

June 25-28, 2025 Hong Kong

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Call for Papers

The 2025 IEEE International Conference on Quantum Control, Computing, and Learning (IEEE qCCL 2025) will be held during June 25-28, 2025, Hong Kong. Accepted papers that meet IEEE Xplore's scope and quality requirements will be sent for being included in IEEE conference proceedings and potentially indexed by EI Compendex. The conference also set Best Paper Award, Best Student Paper Award and Best Poster Award. Details can be found on the conference website https://events.polyu.edu.hk/IEEE_qCCL2025.

This conference will convene leading experts and researchers from a diverse array of scientific disciplines. Attendees will have the opportunity to share their latest research findings and delve into the most cutting-edge developments in the field of quantum technologies. This event also serves as a valuable platform for fostering scientific collaboration and academic exchange among colleagues across quantum control, computing, and learning communities.

Submission and Registration Methods

The conference website is: https://events.polyu.edu.hk/IEEE_qCCL2025
Submission of paper or abstracts: Please follow the instructions on the conference website for submission.
For information related to conference registration, please refer to the conference website.

Topics of interest

Topics of interest for submission include, but are not limited to

1. **Quantum control**
   Quantum coherent control
   Quantum optimal control
   Quantum robust control
   Measurement-based quantum feedback
   Quantum coherent feedback control
   Controllability and observability
   Quantum feedback networks
   Quantum linear system theory
2. **Quantum estimation**
   Quantum filtering
   Quantum smoothing
   Quantum state tomography
   Quantum detector tomography
   Quantum process tomography
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   Quantum system identification
3. **Quantum machine learning**
   Quantum neural networks
   Quantum reinforcement learning
   Quantum kernel learning
   Quantum supervised Learning
   Quantum learning theory
   Quantum classifier
4. **Quantum computing**
   Quantum error correction
   Quantum algorithms
   NISQ quantum computing
   Hybrid quantum-classical algorithms
   Quantum software
   Quantum computing architecture
   Variational quantum circuits
5. **Quantum sensing**
   Quantum sensing protocols
   Quantum metrology
   Quantum precision measurement
   Quantum spin sensor
   Imaging and spectroscopy
   Quantum radar
   Quantum-enhanced imaging
6. **Applications**
   Energy systems
   Finance systems
   Engineering systems
   Numerical optimization
   Physical chemistry
   Material design
   Medicine and healthcare

Important dates

30 September 2024	**Conference submission open**
15 December 2024	Invited session proposal due
15 February 2025	Paper/Abstract submission deadline
15 March 2025	Acceptance notification
15 April 2025	Final paper submission
1 June 2025	The last day of Early Registration
25-28 June 2025	Conference dates

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Quantum computing and control

This talk

Using **concepts and methods from control theory** to better understand and improve quantum computing:

- Key concepts of quantum computing
- Robustness of quantum algorithms

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Quantum Computing Through the Lens of Control

A TUTORIAL INTRODUCTION

JULIAN BERBERICH and DANIEL FINK

Quantum computing is the science of storing and processing information using systems that obey the laws of quantum mechanics [1]. Quantum mechanics describes nature on tiny scales, where it behaves radically different from our everyday experience. On the atomic scale, systems exhibit counterintuitive effects, such as entanglement (a strong form of coupling) or inherent and irresolvable uncertainty [2]. It was first proposed in the 1980s by Richard Feynman that these effects could possibly be exploited to perform computations in a way that is superior to classical computing [3].

Soon after the first inception of quantum computers, algorithms were developed that provably solve certain problems faster than any known classical algorithm. For example, Grover's algorithm [4] can be used to solve unstructured search problems over N elements with a complexity of only $O(\sqrt{N})$. Perhaps most prominently, Shor's

quantum computing that may provide possible speedups over classical algorithms. In particular, quantum simulation is inherently difficult for a classical computer and has, in fact, inspired the concept of a quantum computer in the first place [1, Sect. 4.7], [3]. These early theoretical successes have

Introduction

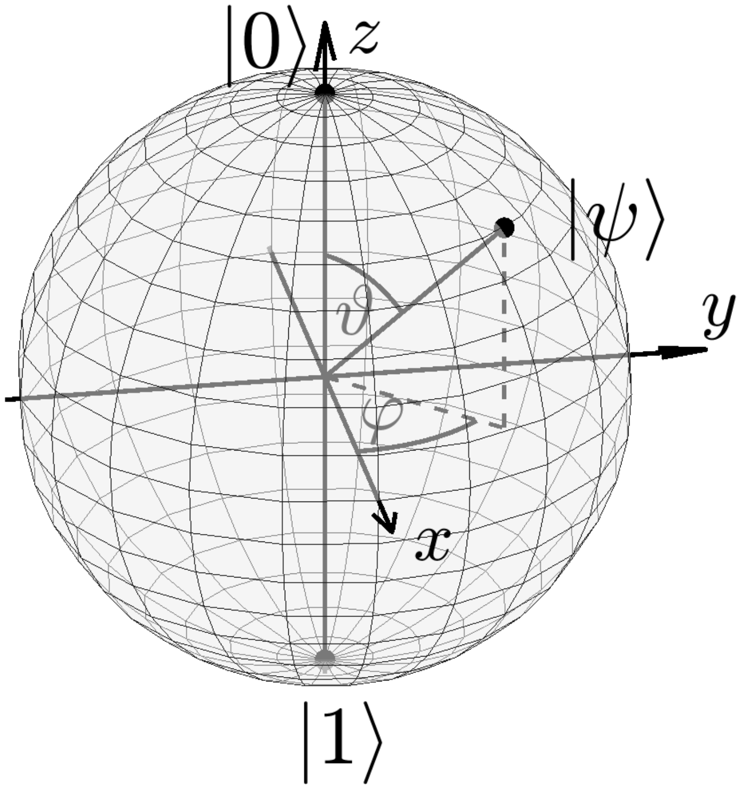
Qubits

Qubit state $|\psi\rangle$ is described by a **unit vector** in $\mathbb{C}^2$

- **Bra-ket** notation $|\psi\rangle$
- Qubits lie in **superposition**: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

  with $\alpha, \beta \in \mathbb{C}$ and $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$
- States differing by global phase $e^{-i\varphi}$ are equivalent

  →Space of qubit states ≅ Bloch sphere



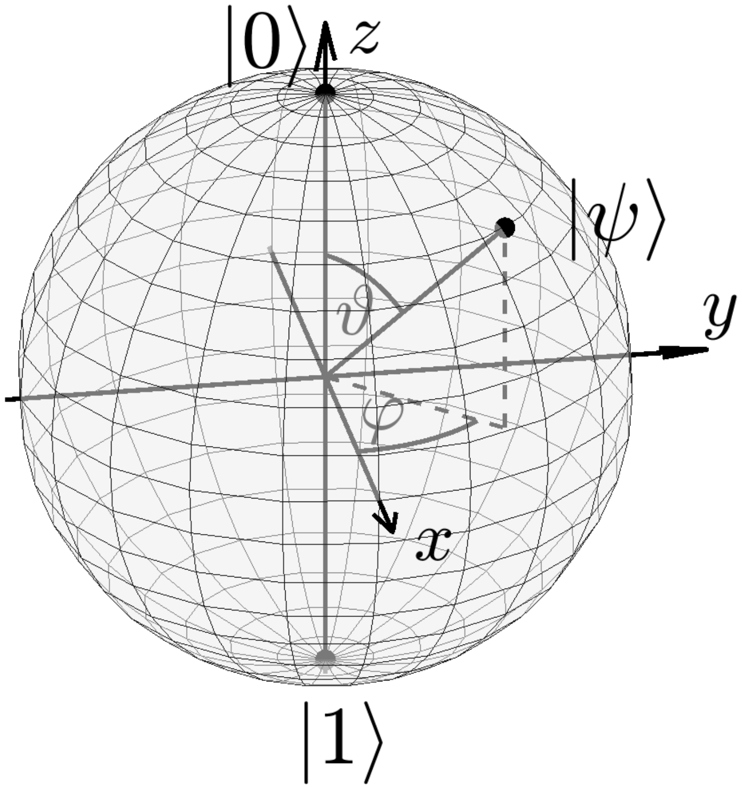
Multi-qubit quantum states

Collection of *n* qubits: vector in $\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 = \mathbb{C}^{2^n}$

Measurement of a quantum state

Projective Measurements

- Taken w.r.t. an **observable** $M = M^\dagger$
  Spectral theorem: $M = \sum_i \lambda_i P_i$ with $P_i = v_i v_i^\dagger$
- **Measurement outcome:** eigenvalue λ_i of M with probability $\langle\psi|P_i|\psi\rangle \coloneqq \psi^\dagger P_i \psi$
- **Collapse of quantum state:** When measuring λ_i, the state after measurement is v_i



Measuring a single qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ w.r.t. Pauli matrix $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Measurement returns 1 with prob. $|\alpha|^2$ and -1 with prob. $|\beta|^2$.
- State after measurement is $|0\rangle$ or $|1\rangle$, respectively.

Measurement of a quantum state

Projective Measurements

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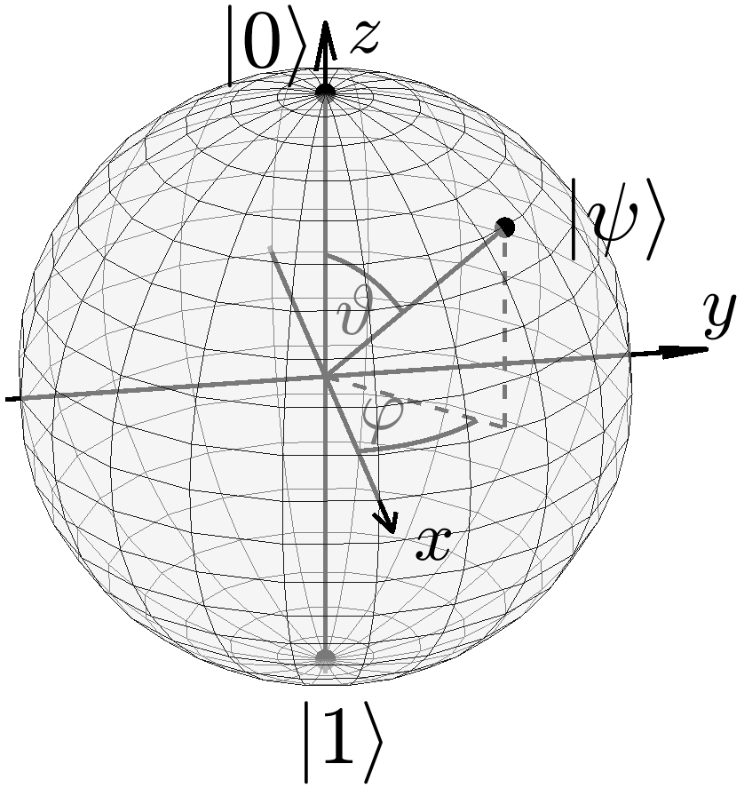


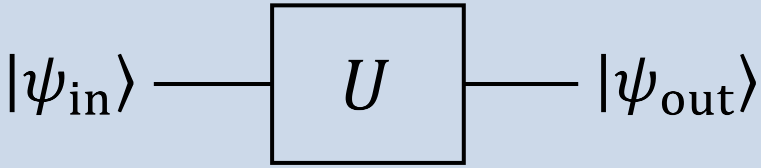
- Repeated measurements allow to evaluate $\langle\psi|M|\psi\rangle$

Measurement can be viewed as evaluation of a **quadratic form!**

Quantum gates

Quantum gates are described by **unitary matrices:**

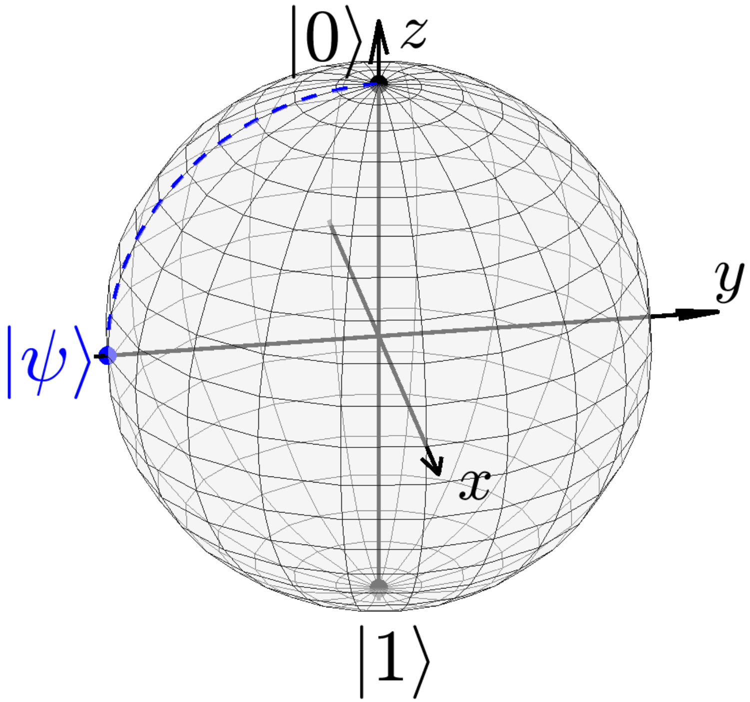
$$|\psi_{\text{out}}\rangle = U|\psi_{\text{in}}\rangle$$



for some $U \in \mathbb{C}^{2^n \times 2^n}$ with $U^\dagger U = I$.

- Quantum gates are **linear** and **reversible**
- Single-qubit gate = **rotation on Bloch sphere**
- **Examples:** Pauli gates

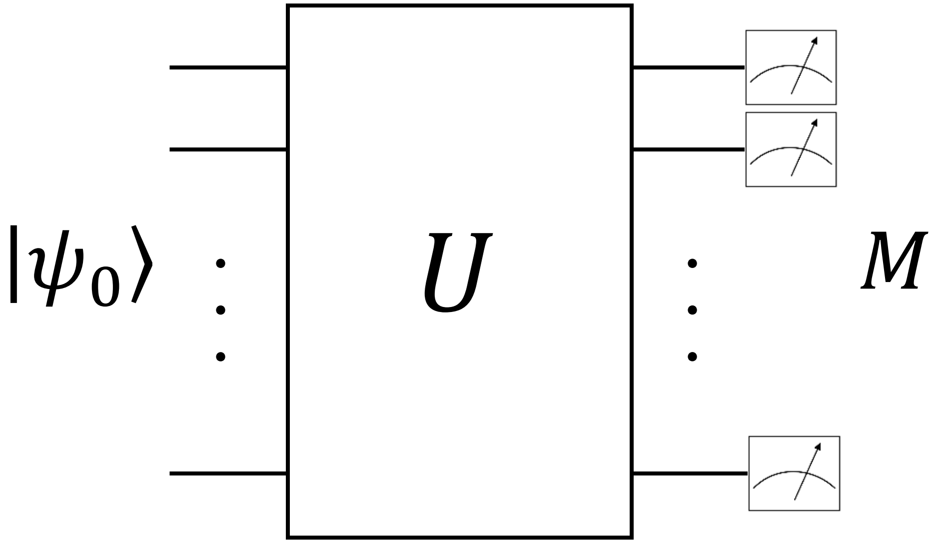
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Ingredients of a quantum algorithm

- Input state $|\psi_0\rangle$
- Unitary matrix U
- Measurement w.r.t. observable M

Summarized in one formula: $\langle\psi_0|U^\dagger M U|\psi_0\rangle$



- Quantum algorithms ≈ linear algebra
- U consists of elementary gates. **Key challenge:** how to choose them

Examples of quantum algorithms

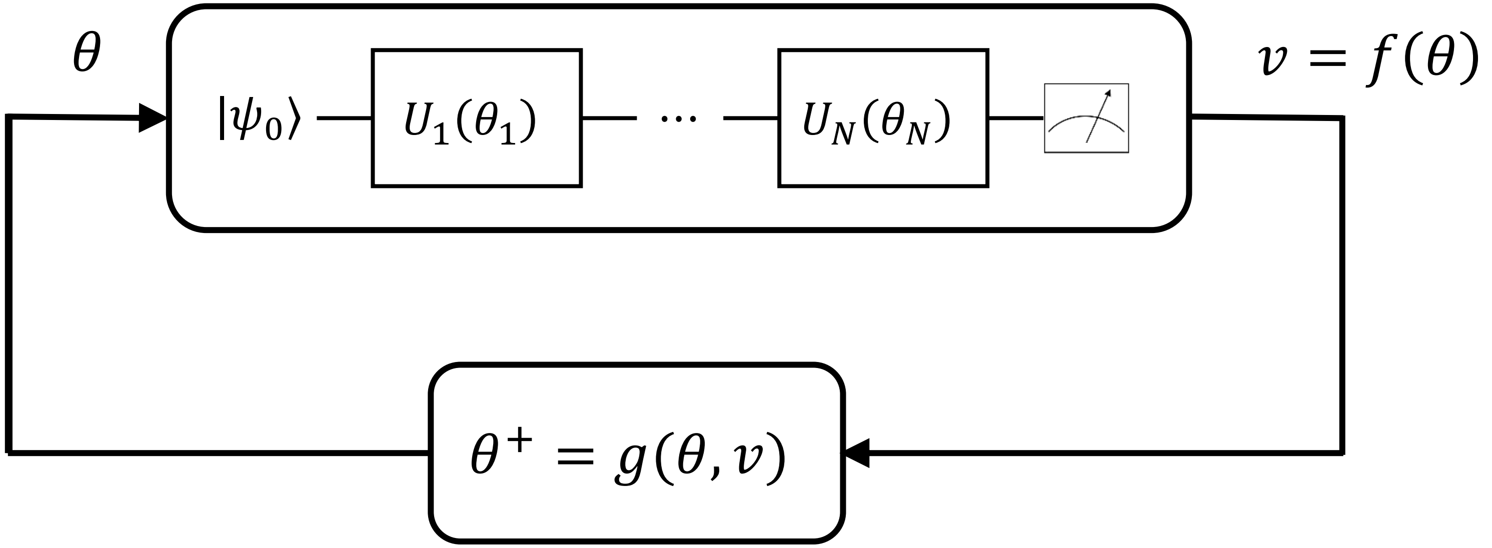
- **Shor's algorithm:** Integer factorization
  - Polynomial complexity
  - Implications for cryptography
- **Quantum simulation:** Solving the Schrödinger equation $|\dot{\psi}\rangle = -iH|\psi\rangle$
  - Classically challenging due to exponential size of $|\psi\rangle$
  - Applications: quantum chemistry, drug design, material science, …
- **Quantum Fourier transform:** Discrete Fourier transform of $|\psi\rangle$
  - Exponential speedup but result is stored in quantum state
  - Key ingredient of other algorithms
- **Grover's algorithm:** unstructured search
  - Quadratic speedup ($O(\sqrt{N})$ instead of $O(N)$)

Variational quantum algorithms

- Any unitary matrix can be written as $U = e^{-iH}$ for some $H = H^\dagger$
- Parametrized unitaries $U(\theta) = U_1(\theta_1) \cdots U_N(\theta_N)$ with $U_j(\theta_j) = e^{-i\theta_j H_j}$.

Goal: minimize $f(\theta)$ with
$f(\theta) = \langle\psi_0|U(\theta)^\dagger M U(\theta)|\psi_0\rangle$

→ Iteratively adapt parameters



Applications of VQAs:

- **Finding minimum eigenvalue of M:** relevant in quantum chemistry
- Combinatorial optimization
- **Machine learning:** Use $f(\theta)$ as function approximator

Variational quantum algorithms

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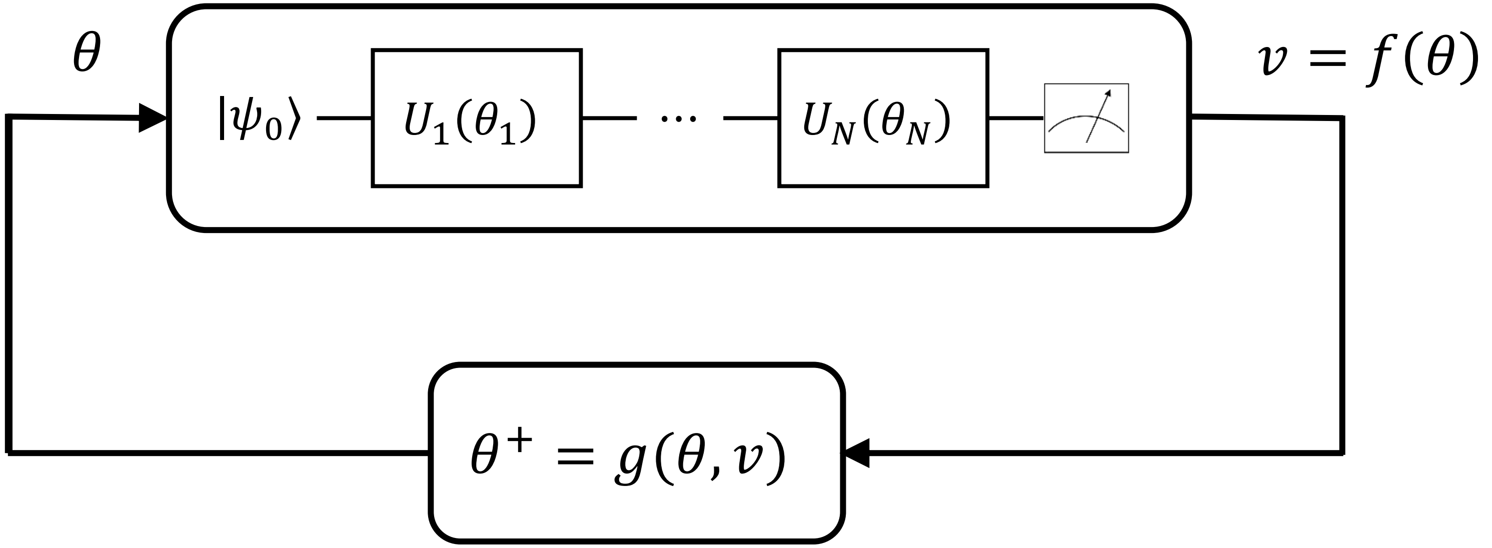

Variational quantum algorithms: Feedback loops of quantum/classical algorithms

- **Lur'e system:** Static nonlinearity + linear dynamical system
- **Challenges:** Non-convexity, optimization landscape, …

Using quantum computers in control

???

Systems and control theory ⇄ Quantum computing

Scope of this talk

Some problems addressed via quantum algorithms:

- Combinatorial optimization
- Mixed-integer optimization
- Semidefinite programming
- Linear systems of equations

2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Using quantum computers in control: interval matrix properties

Jan Schneider, Julian Berberich

Quantum computing has potential to address computational problems in control

Physical computation

Connections

Quantum annealing in the transverse Ising model

Tadashi Kadowaki and Hidetoshi Nishimori
Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan
(Received 30 April 1998)

Quantum Computation over Continuous Variables

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MIT Department of Mechanical Engineering, MIT 3-160, Cambridge, Massachusetts 02139

Samuel L. Braunstein
SEECS, University of Wales, Bangor LL57 1UT, United Kingdom
(Received 27 October 1998)

Quantum neuromorphic computing EP

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Neuromorphic quantum computing

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Institute for Theoretical Physics, Heidelberg University, Philosophenweg 16, 69120 Heidelberg, Germany

Ising problems in quantum computing

Ising problem: minimize the Hamiltonian of a set of n spins $x_i = \pm 1$

$$H(x) = -\sum_{i<j} J_{ij} x_i x_j - \sum_i h_i x_i$$

Quantum reformulation

Steer quantum state $|\psi\rangle$ to minimum of $\langle\psi|M|\psi\rangle$ with observable

$$M = -\sum_{i<j} J_{ij} Z_i Z_j - \sum_i h_i Z_i$$

- **Notation** Z_j: Applying $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to the j-th qubit
- Encompasses **combinatorial optimization problems**

Tackling Ising problems via VQAs

Minimize $f(\theta) = \langle \psi_0 | U(\theta)^\dagger M U(\theta) | \psi_0 \rangle$

with $M = -\sum_{i<j} J_{ij} Z_i Z_j - \sum_i h_i Z_i$

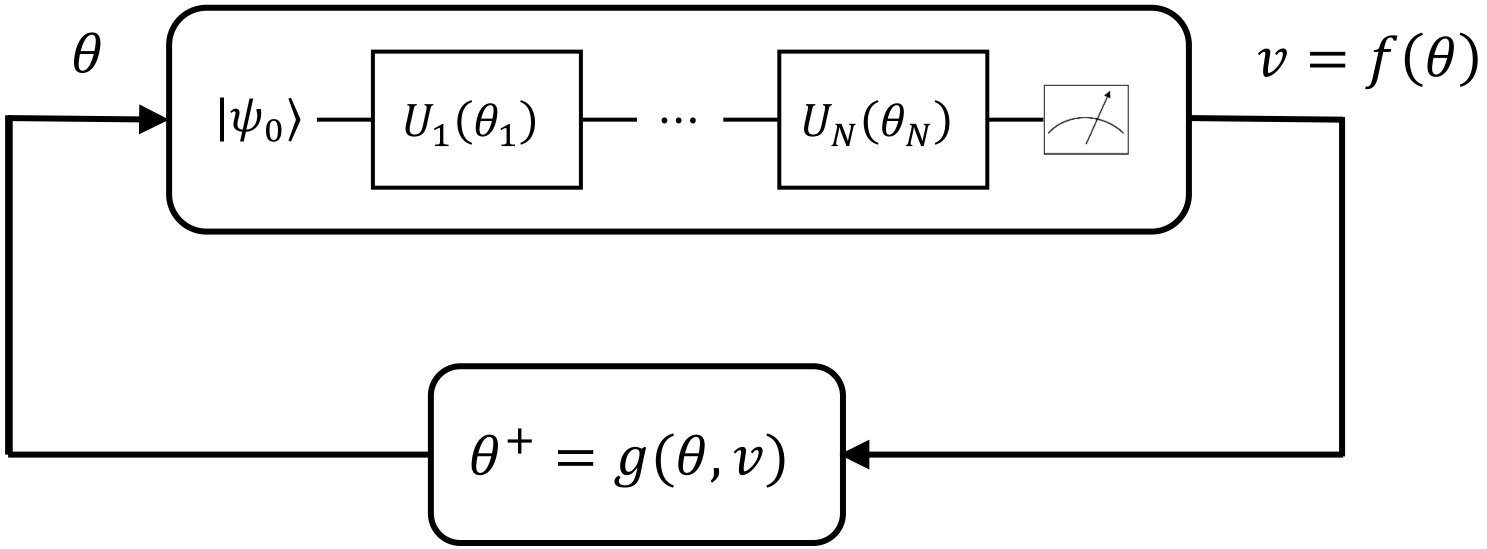
A Quantum Approximate Optimization Algorithm

Edward Farhi and Jeffrey Goldstone

Center for Theoretical Physics

Massachusetts Institute of Technology

Cambridge, MA 02139



- Choice of $U(\theta)$?
- Optimization?
- Guarantees?

Quantum annealing

Schrödinger equation: $|\dot{\psi}(t)\rangle = -iH(t)|\psi(t)\rangle$

Quantum annealing

Evolve with Hamiltonian $H(t) = (1 - u(t))B + u(t)M$ with $B = \sum_j X_j$

1. Initial state $|\psi(0)\rangle = |+\rangle^{\otimes n} = [1 \quad \dots \quad 1]^{\mathrm{T}}$
2. Change input **slowly** from $u(0) = 0$ to $u(T) = 1$
3. Final state $|\psi(T)\rangle$

- **Adiabatic theorem:**
  $|\psi(T)\rangle$ is the ground state of M if the system evolves **sufficiently slowly**.
- Otherwise: used as a **heuristic**.

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⋮

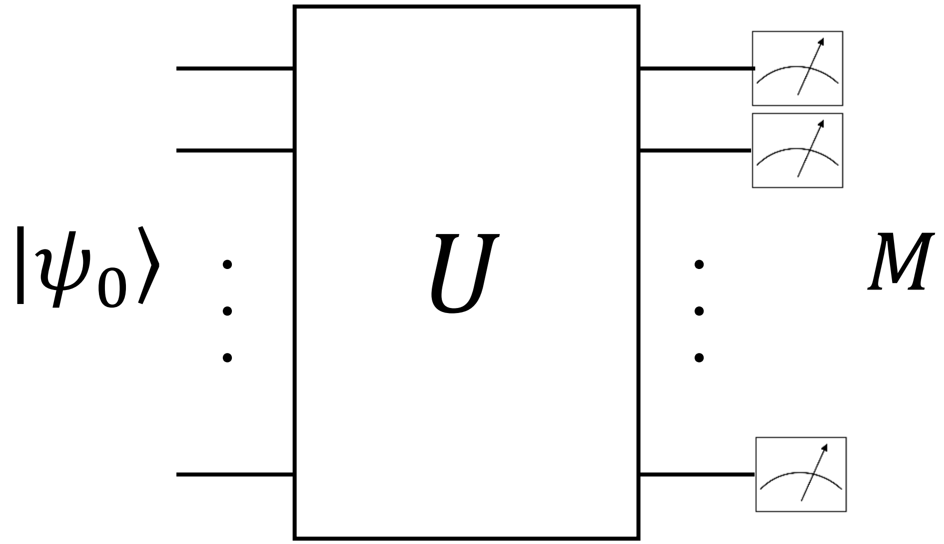
?

Robustness of quantum algorithms: Coherent control errors

Quantum errors

Errors can enter at any stage:

- Preparing the input state $|\psi_0\rangle$
- Implementing the gates in U
- Performing the measurement M
- Keeping the qubits in **coherence**



Quantum error correction: detect & correct errors via additional gates & qubits
Problem: possibly large overhead

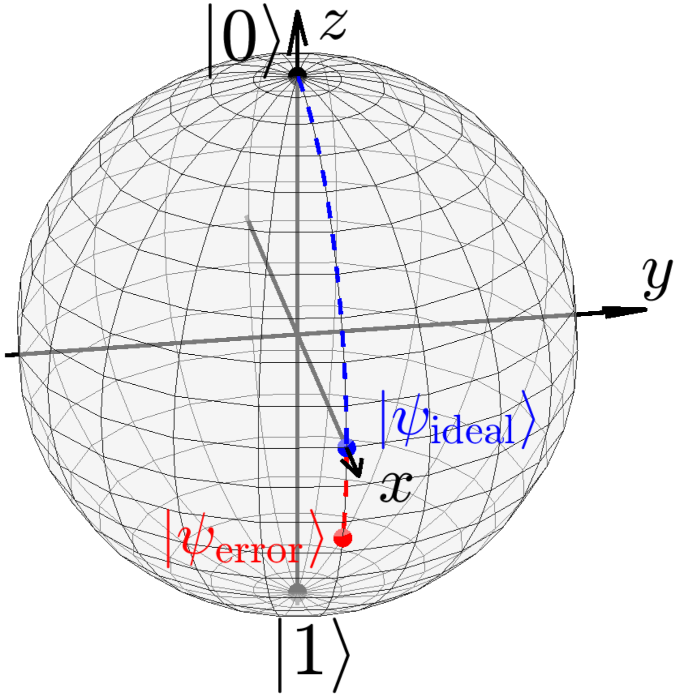
Understanding of errors and **how to mitigate them** is active research topic!

Quantum errors

Coherent control errors

- Any unitary matrix U can be written as $U = e^{-iH}$ for some $H = H^\dagger$
- **Coherent control error:** The **ideal** gate $U_{\mathrm{ideal}} = e^{-iH}$ is replaced by $U_{\mathrm{noisy}}(\varepsilon) = e^{-i(1+\varepsilon)H}$ for some $\varepsilon \in \mathbb{R}$

- Caused by imprecise control, e.g., miscalibration
- Similar results for **more general errors** (coherent & incoherent)



Our results: Analysis of coherent control errors

- Study inherent robustness of quantum algorithms
- Implications for algorithm design

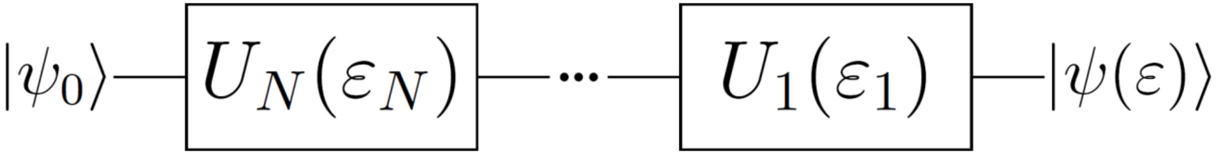
Problem setup

Ideal quantum algorithm

$|\hat{\psi}\rangle = \hat{U}_1 \cdots \hat{U}_N |\psi_0\rangle$ with $\hat{U}_j = e^{-iH_j}$

Noisy quantum algorithm

$|\psi(\varepsilon)\rangle = U_1(\varepsilon_1) \cdots U_N(\varepsilon_N)|\psi_0\rangle$ with
$U_j(\varepsilon_j) = e^{-i(1+\varepsilon_j)H_j}$ and $\varepsilon_j \in \mathbb{R}$



- They are related via $|\hat{\psi}\rangle = |\psi(0)\rangle$
- **Assumption:** ε is bounded, i.e., $\|\varepsilon\| \leq \bar{\varepsilon}$

Problem: Robustness analysis

Find **fidelity lower bound**: $|\langle\psi(\varepsilon)|\hat{\psi}\rangle| \geq 1 - c\bar{\varepsilon}^2$ for some $c > 0$.

Robustness analysis

Key idea: use concept of **Lipschitz bounds**

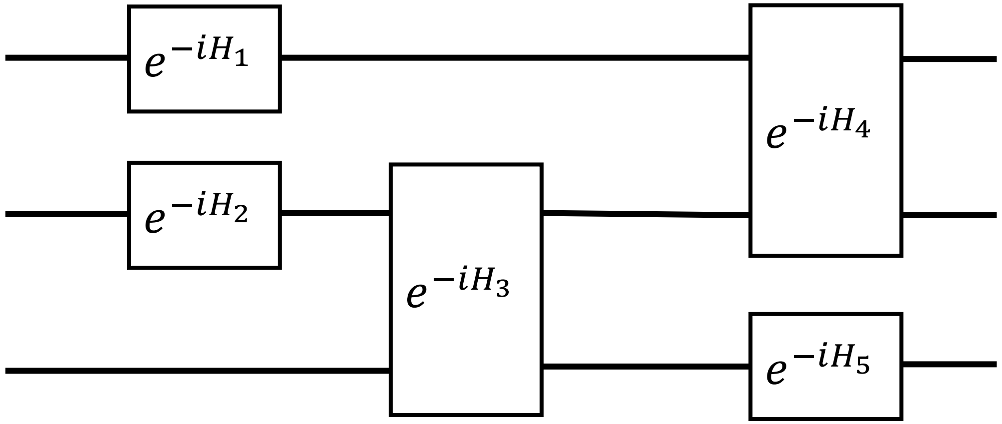
Definition: $L > 0$ is a **Lipschitz bound** of $|\psi\rangle$ if

$$\||\psi(\varepsilon)\rangle - |\psi(\varepsilon')\rangle\| \leq L\|\varepsilon - \varepsilon'\| \quad \text{for all } \varepsilon, \varepsilon' \in \mathbb{R}^N.$$

Theorem

$L = \sum_{j=1}^{N} \|H_j\|$ is a Lipschitz bound of $|\psi\rangle$.

Example: $L = \|H_1\| + \cdots + \|H_5\|$



Robustness analysis

Corollary

For any ε with $||\varepsilon|| \leq \bar{\varepsilon}$ and any initial state $|\psi_0\rangle$, it holds that

$$\left|\langle\psi(\varepsilon)|\hat{\psi}\rangle\right| \geq 1 - \left(\sum_{j=1}^{N} \left\|H_j\right\|\right)^2 \frac{\bar{\varepsilon}^2}{2}.$$

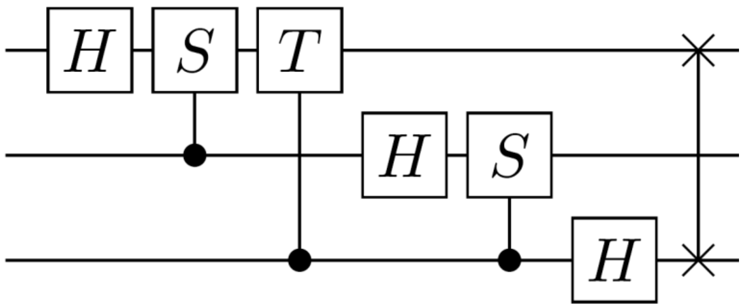
- **Fidelity loss bounded** by $||H_j||$ and noise bound $\bar{\varepsilon}$
- Smaller $||H_j||$ → better robustness

Design of the algorithm influences its robustness!

Application: Quantum Fourier Transform

We consider five different implementations of the Quantum Fourier Transform. For each, we

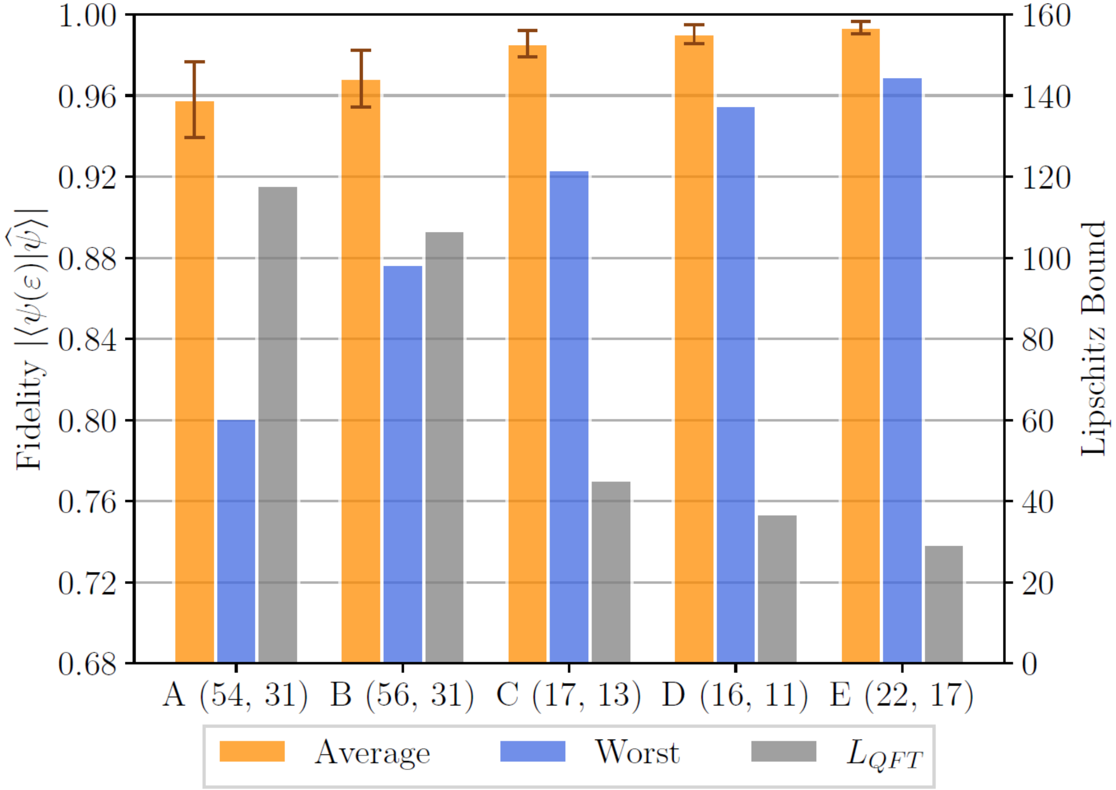
- **compute** the Lipschitz bound $\sum_{j=1}^{N} \|H_j\|$
- **simulate** the circuit with coherent control errors.



Discussion

- Small $\sum_{j=1}^{N} \|H_j\|$ → high fidelity
- Existing metrics such as gate count or depth **do not explain the outcome**

Framework provides a priori **robustness guarantees!**



Robustness of quantum algorithms: Coherent errors

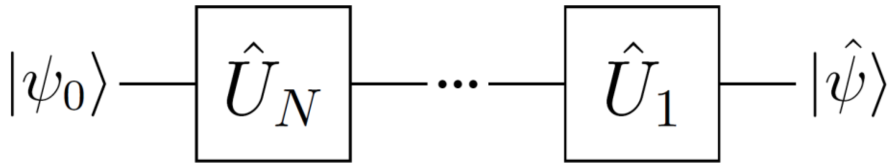
Problem setup

Ideal quantum algorithm

$|\hat{\psi}\rangle = \hat{U}_1 \cdots \hat{U}_N |\psi_0\rangle$ with $\hat{U}_j = e^{-iH_j}$

Noisy quantum algorithm

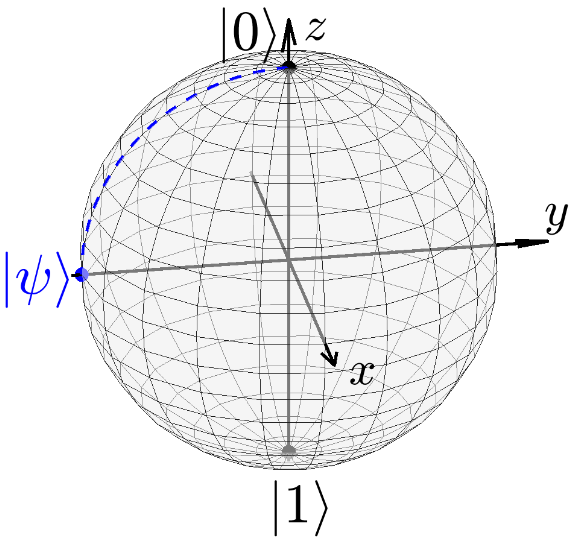
$|\psi\rangle = U_1 \cdots U_N |\psi_0\rangle$ with noisy gates U_j



- Noisy gates: $U_j = \hat{U}_j U_{\mathrm{e},j}$ with error $U_{\mathrm{e},j}$ → coherent error

Assumption: Error model

$U_{\mathrm{e},j} = e^{-iH_{\mathrm{e},j}}$ with $H_{\mathrm{e},j} \in \mathcal{H}_{\mathrm{e},j}$ for known set $\mathcal{H}_{\mathrm{e},j}$



- Examples:
  - Rotation with unknown but bounded angle
  - Coherent control errors

Theoretical analysis

Goal: For a given algorithm and error model, compute a **fidelity bound**

Definition: $G = \frac{1}{N}\sum_{j=1}^{N} \hat{V}_j^\dagger H_{\mathrm{e},j}\hat{V}_j$ where $\hat{V}_j$ depends on the algorithm

Theorem

If $\|H_{\mathrm{e},j}\| \leq \delta$ and $\|G\| \leq \gamma\delta$ for any $H_{\mathrm{e},j} \in \mathcal{H}_{\mathrm{e},j}$, then

$$|\langle\psi|\hat{\psi}\rangle| \geq 1 - \frac{1}{2}\delta^2 N^2 \left(\gamma + \frac{N-1}{2}\delta\right)^2$$

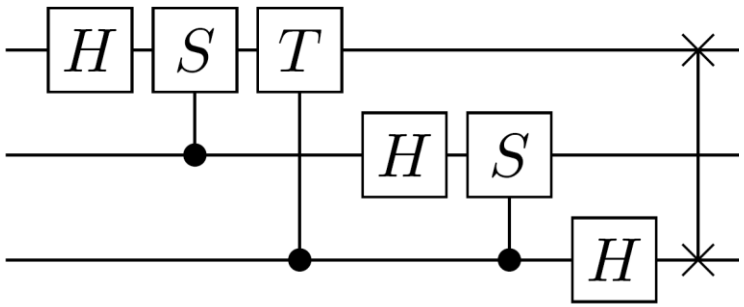
- Bound γ is explicitly computable
- Algorithm-dependent vs. intrinsic error

Guideline for algorithm design: minimize γ

Application: Quantum Fourier Transform

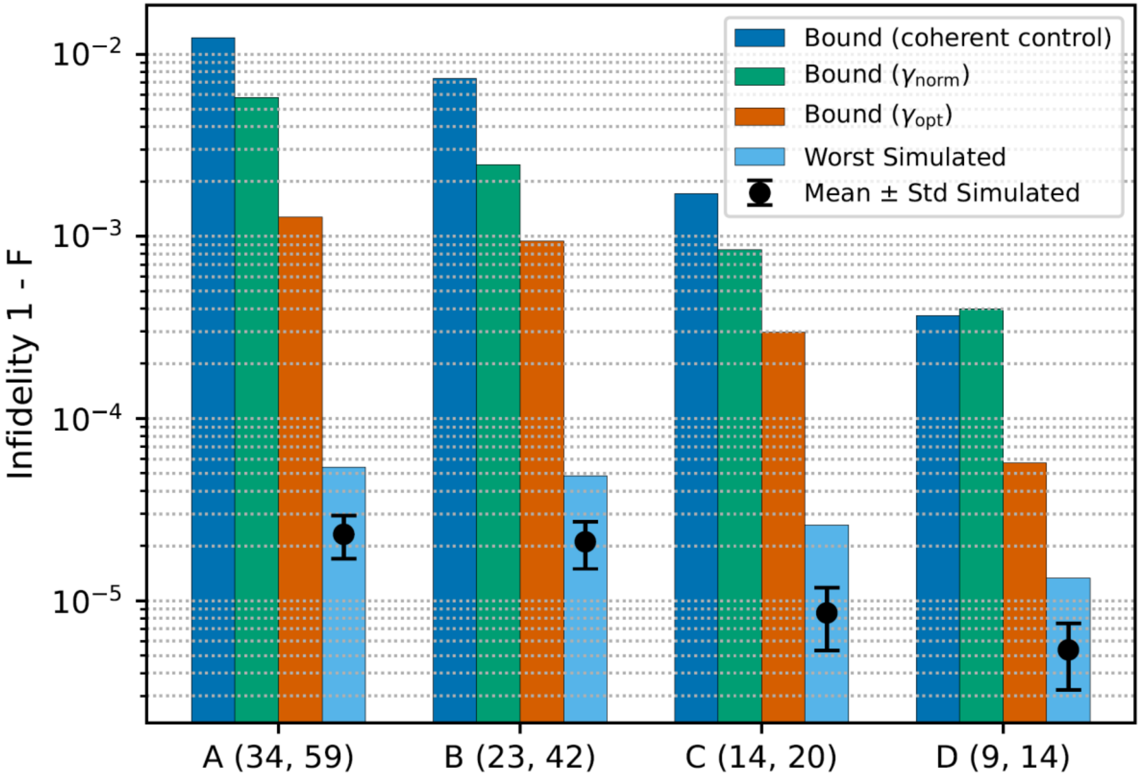
We consider different implementations A-D of the Quantum Fourier Transform. For each, we

- **compute** the fidelity bound with different error models (coherent control error)



Discussion

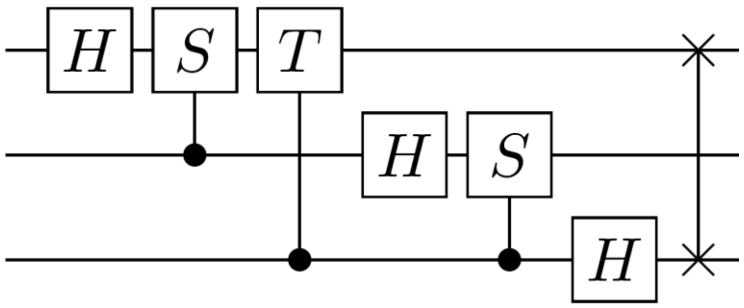
- **Bounds correlate with simulation results**
- **Results are tighter** than prior work

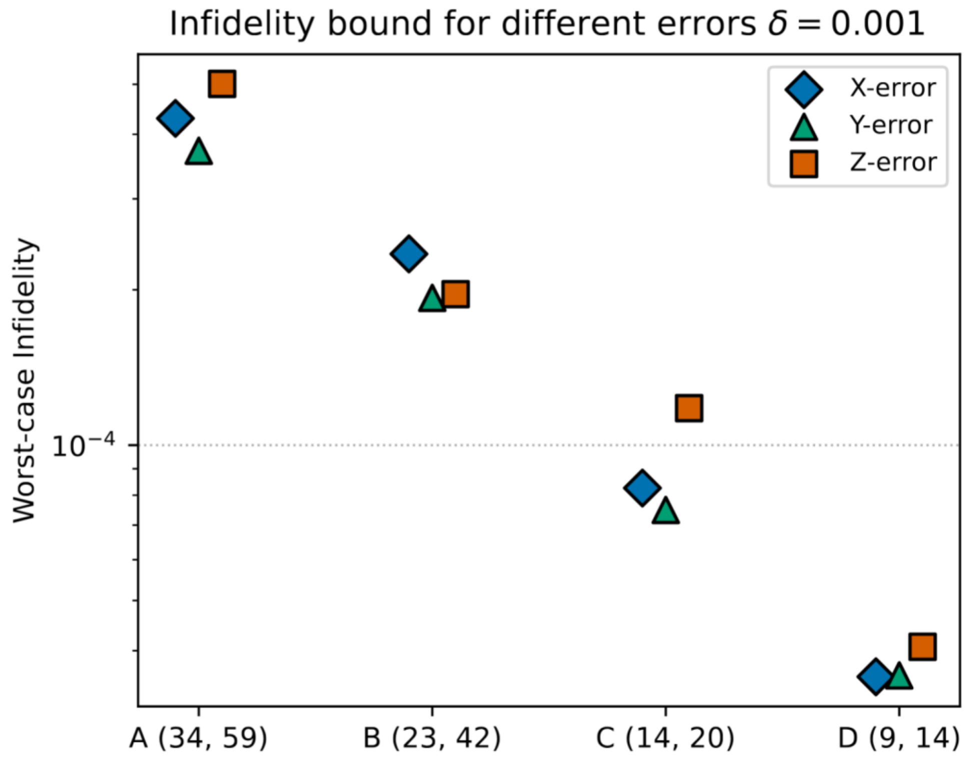


Application: Quantum Fourier Transform

We consider different implementations A-D of the Quantum Fourier Transform. For each, we

- **compute** the fidelity bound with different error models (X/Y/Z-rotations)





Discussion

- **Robustness properties are non-trivial** and depend on the algorithm + error model
- Works for **arbitrary coherent errors**

Flexible framework for robustness analysis of quantum algorithms

Conclusion

Wrap-up

Quantum algorithms consist of:

- Qubit state = complex unit vector
- Quantum gate = unitary matrix → rotation
- Measurement = quadratic form

Wrap-up

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Quantum Computing Through the Lens of Control

A TUTORIAL INTRODUCTION

JULIAN BERBERICH and DANIEL FINK

Quantum computing is the science of storing and processing information using systems that obey the laws of quantum mechanics [1]. Quantum mechanics describes nature on tiny scales, where it behaves radically different from our everyday experience. On the atomic scale, systems exhibit counterintuitive effects, such as entanglement (a strong form of coupling) or inherent and irresolvable uncertainty [2]. It was first proposed in the 1980s by Richard Feynman that these effects could possibly be exploited to perform computations in a way that is superior to classical computing [3].

Soon after the first inception of quantum computers, algorithms were developed that provably solve certain problems faster than any known classical algorithm. For example, Grover's algorithm [4] can be used to solve unstructured search problems over N elements with a complexity of only $O(\sqrt{N})$. Perhaps most prominently, Shor's

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quantum computing that may provide possible speedups over classical algorithms. In particular, quantum simulation is inherently difficult for a classical computer and has, in fact, inspired the concept of a quantum computer in the first place [1, Sect. 4.7], [3]. These early theoretical successes have

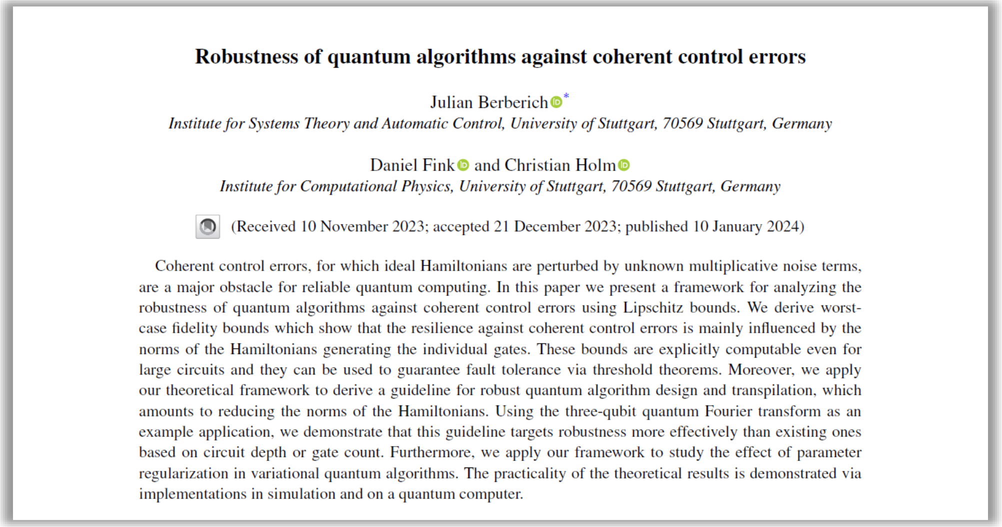
Wrap-up

Quantum algorithms consist of:

- Qubit state = complex unit vector
- Quantum gate = unitary matrix → rotation
- Measurement = quadratic form

Robustness of quantum algorithms: Coherent control errors

- Worst-case bounds via Lipschitz continuity
- Design guidelines: small $\|H_j\|$


Robustness of quantum algorithms against coherent control errors

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Coherent control errors, for which ideal Hamiltonians are perturbed by unknown multiplicative noise terms, are a major obstacle for reliable quantum computing. In this paper we present a framework for analyzing the robustness of quantum algorithms against coherent control errors using Lipschitz bounds. We derive worst-case fidelity bounds which show that the resilience against coherent control errors is mainly influenced by the norms of the Hamiltonians generating the individual gates. These bounds are explicitly computable even for large circuits and they can be used to guarantee fault tolerance via threshold theorems. Moreover, we apply our theoretical framework to derive a guideline for robust quantum algorithm design and transpilation, which amounts to reducing the norms of the Hamiltonians. Using the three-qubit quantum Fourier transform as an example application, we demonstrate that this guideline targets robustness more effectively than existing ones based on circuit depth or gate count. Furthermore, we apply our framework to study the effect of parameter regularization in variational quantum algorithms. The practicality of the theoretical results is demonstrated via implementations in simulation and on a quantum computer.


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Wrap-up

Quantum algorithms consist of:

- Qubit state = complex unit vector
- Quantum gate = unitary matrix → rotation
- Measurement = quadratic form

Robustness of quantum algorithms: Worst-case fidelity bounds and implications for design

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Robustness of quantum algorithms: Coherent and incoherent errors

- Worst-case bounds & design guidelines
- Details in the paper

Next steps: application, tighter bounds, …

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